Curvatures of Productivity, Elasticities of the Output and Stages of Production

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Abstract: *As a rule, the concept of marginal productivity is used in order to determine the efficient distribution of incomes or optimal levels for inputs allocations in the context of diminishing marginal returns. But the definition of the concept of the endogenous economic growth imposed the relaxation of assumptions on the feature of the marginal returns, admitting that it is possible, in certain situations, to deal with both increasing and decreasing marginal returns. This paper defines a neoclassical production function with one input, which admits both the increasing and decreasing marginal returns and, in certain manner, the existence of the Jevons paradox. The respective production function is then used for the analysis of the curvature of the marginal and average productivity and of the elasticity of the output. On this basis, we are able to reconfirm the neoclassical assumptions on the size of the elasticity of the output in the context of the decreasing marginal returns and output maximization, on the one hand, and to show that in the context of a convex curvature and of increasing of both average and marginal productivity the elasticity of the output is higher than 2, on the other hand. Also, it is propsed a redefinition of the notion of stages of production considering the features of the curvature of the average productivity and of the elasticity of the output.*

Keywords: *non-constant marginal returns, third derivative, neoclassical production function, efficiency parameter, Jevons paradox, elasticity of the output, inflection point, stages of production*

JEL Classification: *B13, C02, D21, D24, J24*

1. Introduction

In the history of economic theory, the definition of the concept of “marginal productivity” represents an important turning point. A forerunner of the marginality approach of the analysis of economic and social phenomena was Daniel Bernoulli. He studied the

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Some ideas related to the concept of marginal productivity can be found in David Ricardo's works. But the first definition of the principle of marginal productivity and its application to factor pricing was given by J.H. von Thune in 1826. During the first half of 19-th century, in Europe, important contributions to the development of marginal analysis applied to economic activity were brought by A. Cornet, J.Dupuit and H. H. Gossen.

During the second half of 19-th century neo-classical economists paid a special attention to definition of marginal computation in economic theory. In fact, the respective neo-classical economic paradigm may be viewed as a "marginal revolution". Important contributions to the development of concepts of marginal utility or marginal productivity and their use in economic analysis were brought by W.S. Jevons, A. Marshall, J.B. Clark, and K. Wicksell. In 20-th century, J.R. Hicks (1932), enriched the concept of marginal productivity by relating it to wages determination in the context of interaction between supply and demand in a situation of competitive equilibrium of a free market.

Marginal productivity theory is applied in the context of a static world characterized by a perfect competition, homogenous and divisible inputs, and decreasing returns to scale. Also, there is made the assumption that inputs can be easily substituted. Under these conditions, it is possible to determine the level of output which permits to maximize the profit of entrepreneur and to remunerate all the inputs at correct prices, which are equal with the their marginal productivity.

Ones of the assumptions of marginal productivity theory are subject of criticism because some economists consider that there are assumptions which are not confirmed by real economic experiences. The most frequent critiques are related to: a) the homogeneity and perfect divisibility of inputs, b) perfect substitution between inputs, c) full employment of inputs, d) accurate measurement of marginal product and the continuity and derivability of production function, especially in the region where the marginal productivity is presumed to be equal with zero, e) the law of diminishing returns is not applicable in all economic situations.

The criticism of some aspects of theory of marginal productivity may not block the investigation of cognitive valences of the above-mentioned theory. A possibility to reveal respective cognitive valences is to make efforts to identify a new form of production function, which is compatible with the requirements of standard theory of marginal productivity.

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2. Identification of production function with one input in the context of decreasing marginal returns

The assumption that in economic activity the firms or other economic and social entities deal with decreasing marginal returns implies that the curvature of the output related to input allocation is a concave one. In other words, if we note the output level with Y and allocated quantity from the considered input with T, we may write:

$$\frac{d^2Y}{d^2T}=r< 0 \qquad (1)$$

where:

$\frac{d^2Y}{d^2T}$= second derivative of output related to considered input.

The formula (1) implies that the second derivative of the output related to the considered input is constant and the third derivative ($\frac{d^3Y}{d^3T}$) is equal to zero. The respective situation is assumed in the definition of the most frequently used production functions such as Cobb-Douglas or CES.

We notice that the first derivative of the output related to the considered production function is the marginal productivity

Therefore, the marginal productivity related to considered input ($\frac{dY}{dT}$) can be written as:

$$\frac{dY}{dT} = q + r \cdot T \qquad (2)$$

In this context, the relationship between the level of output and the considered input is:

$$Y = p + q \cdot T + \frac{r \cdot T^2}{2} \qquad (3)$$

If we analyse the formula (3) and we treat the respective formula as a production function with only one input we may find that it is necessary that p=0, due to the implementation of the principle that in a production function the output is obtained only if a certain quantity of input is consumed.

If we use the following notations: $a = q$ and $b = -\frac{r}{2}$, with a>0 and b >0, we are able to express the above-mentioned production function as:

$$Y = a \cdot T - b \cdot T^2 \tag{4}$$

equivalent to:

$$Y = T \cdot (a - b \cdot T) \tag{5}$$

Formula (4) allows us to compute the maximum quantity of input which may be allocated in order to obtain a certain quantity of output. We may accept the assumption that it is possible to employ the considered input in all situations when it is possible to obtain: Y>0.

The condition Y=0 is fulfilled if $T = \frac{a}{b}$. In other words, the ratio $(\frac{a}{b})$ represents the maximum quantity which can be allocated from the considered input.

The marginal productivity related to considered input is:

$$\frac{dY}{dT} = a - 2b \cdot T \tag{6}$$

Also, we have:

$$\frac{d^2Y}{d^2T} = -2b \tag{7}$$

Because the curvature of the output depending on quantity allocated from the considered input is a concave one, the maximum of output level is obtained if the marginal productivity is equal with zero. The respective situation occurs if: $T = \frac{a}{2b}$.

In other words in case of the production function defined by the formula (4), the maximum output (Ymax) is obtained only if one half of the employable quantity of considered input is used.

It is to mention that

$$Y_{max} = \frac{a^2}{4b} \tag{8}$$

Having in view the form of production function presented in formula (5), we may express the average productivity of considered input ($\frac{Y}{T}$) as:

$$\frac{Y}{T} = a - b \cdot T \quad (9)$$

Consequently, the first derivative of average productivity related to considered input ($\frac{d(\frac{Y}{T})}{dT}$) may be expressed as:

$$\frac{d(\frac{Y}{T})}{dT} = -b \quad (10)$$

Formula (8) shows that average productivity is a decreasing function of the level of quantity allocated from considered input. Also, we notice that the second derivative of the average productivity ($\frac{d^2(\frac{Y}{T})}{d^2T}$) is equal to zero.

3. A production function with one input in the context of relaxation of the assumptions regarding the marginal returns

The assumption of decreasing marginal returns was relaxed in conditions of defining of the concepts of endogenous technical change or endogenous economic growth. It was accepted the fact that returns to scale are not necessary decreasing in conditions of the occurrence and implementation of new technologies such as information and telecommunication technologies in the framework of productive system. The use of respective technologies creates conditions for impressive growth of productivity of inputs, especially in case of labour force.

Therefore, it is recommendable to consider both the cases of increasing and decreasing returns to scale and consequently to modify the form of production function expressed by formula (5).

If we consider the notion of non-constant marginal returns related to a considered input it is possible to generalize the production function defined by the formula (5).

Hence, we deal with a production function which may be defined by the following formula:

$$Y = T^{\alpha} \cdot (a - b \cdot T) \tag{11}$$

where: α= an efficiency parameter acting as a proxy constant elasticity of output in case of Cobb-Douglas production function with one input, because we may write:

$$Y = a \cdot T^{\alpha} \cdot (1 - \frac{b}{a} \cdot T) \tag{12}$$

This way, we can reveal that the production function (11) adopts some assumptions related to the efficiency of the use of the production factors like in case the Cobb-Douglas production function. The difference between the production function (11) and the Cobb-Douglas production function is that the first admits a maximum level of the output, while the latter is continuously increasing.

We may observe that if α=1, we are in the case of the production function defined by formula (5).

It is to note that the maximum quantity of input which can be allocated remain the same as in case of the production function defined by formula (5).

Marginal productivity related to production function defined by formula (11) can be written as:

$$\frac{dY}{dT} = \alpha \cdot T^{\alpha-1} \cdot (a - b \cdot T \cdot \frac{\alpha+1}{\alpha}) \tag{13}$$

$\frac{dY}{dT} = 0$ if $T = \frac{\alpha}{\alpha+1} \cdot \frac{a}{b}$

Hence, the maximum level of the output is:

$$Y_{max} = \frac{a}{(1+\alpha)} \cdot (\frac{a}{b})^{\alpha} \cdot (\frac{\alpha}{1+\alpha})^{\alpha} \tag{14}$$

We note that the marginal productivity is positive if $T < \frac{\alpha}{\alpha+1} \cdot \frac{a}{b}$ and negative if $T > \frac{\alpha}{\alpha+1} \cdot \frac{a}{b}$. In other words, the output grows as the quantity allocated from the considered input increases but is lower than $\frac{\alpha}{\alpha+1} \cdot \frac{a}{b}$.

If $> \frac{\alpha}{\alpha+1} \cdot \frac{a}{b}$., an increase of the allocated quantity of considered input implies the decrease of the output.

The average productivity can be determinated by the formula:

$$\frac{Y}{T} = T^{(\alpha-1)} \cdot (a - b \cdot T) \qquad (15)$$

We notice that the growth of the parameter α determinates the increase of both the allocated quantity of the considered input and the average productivity. This way, we are in a situation near the Jevons paradox, because the increase of the average productivity favours the increase of the demand for the considered input.

4. The curvature of the marginal productivity

The production function defined by the formula (11) allows us to reveal the curvature of the marginal productivity and implicitly the features of the marginal returns

In fact, we need to determine the features of the second and third derivative of output related to considered input.

The second derivative of output related to considered input ($\frac{d^2Y}{d^2T}$) can be expressed as:

$$\frac{d^2Y}{d^2T} = \alpha \cdot (\alpha - 1) \cdot T^{\alpha-2} \cdot (a - b \cdot T \cdot \frac{\alpha+1}{\alpha-1}) \qquad (16)$$

We notice that $\frac{d^2Y}{d^2T} = 0$ if $= \frac{\alpha-1\cdot}{\alpha+1} \cdot \frac{a}{b}$.

The third derivative of output related to considered input ($\frac{d^3Y}{d^3T}$) can be expressed as:

$$\frac{d^3Y}{d^3T} = \alpha \cdot (\alpha - 1) \cdot (\alpha - 2) \cdot T^{\alpha-3} \cdot (a - b \cdot T \cdot \frac{\alpha+1}{\alpha-2}) \qquad (17)$$

We notice that $\frac{d^3Y}{d^3T} = 0$ if $T = \frac{\alpha-2}{\alpha+1} \cdot \frac{a}{b}$

The signs of the expressions (16) and (17) are given both by the size of the parameter α and by the allocated quantity of the considered input. In fact, we deal with two critical values of the parameter α. A first critical value is α1=1, while the second critical value is α2=2[1]. Hence, we deal with three situations from the point of view of the size of the parameter α, when we analyse the impact of the respective parameter on the curvature of the marginal productivity (table 1).

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[1] The situation when α=2, is implicitly considered in F.M. Pavelescu (2003) in order to reveal the share of the wages in the gross domestic product which ensures the equilibrium on the labour market.

Table 1. The signs of the proposed production function and its derivatives depending on the parameter α and quantity allocated from the considered input

Allocated quantity of input	Y	$\frac{dY}{dT}$	$\frac{d^2Y}{d^2T}$	$\frac{d^3Y}{d^3T}$
0<α<1				
T<((α-2)/(α+1)*(a/b))	>0	>0	<0	>0
((α-2)/(α+1)*(a/b))<T<((α-1)/(α+1) *(a/b))	>0	>0	<0	>0
((α-2)/(α+1)*(a/b))<T<(α/(α+1)*(a/b))	>0	>0	<0	>0
T>(α/(α+1) *(a/b))	>0	<0	<0	>0
1<α<2				
T<((α-2)/(α+1)*(a/b))	>0	>0	>0	<0
((α-2)/(α+1)*(a/b))<T<((α-1)/(α+1) *(a/b))	>0	>0	>0	<0
((α-2)/(α+1)*(a/b))<T<(α/(α+1)*(a/b))	>0	>0	<0	<0
T>(α/(α+1) *(a/b))	>0	<0	<0	<0
α>2				
T<((α-2)/(α+1)*(a/b))	>0	>0	>0	>0
((α-2)/(α+1)*(a/b))<T<((α-1)/(α+1) *(a/b))	>0	>0	>0	<0
((α-2)/(α+1)*(a/b))<T<(α/(α+1)*(a/b))	>0	>0	<0	<0
T>(α/(α+1) *(a/b))	>0	<0	<0	<0

If 0<α<1, we deal with a continuous decreasing marginal productivity under a convex curvature. The curvature of output is a concave one. A critical point of the allocated quantity of the considered input is $T = \frac{\alpha}{\alpha+1} \cdot \frac{a}{b}$ when the marginal productivity is equal to zero and the maximum level of the output is obtained.

If 1<α<2, we deal with a concave curvature of the marginal productivity. In this context we identify a second critical point of the allocated quantity of the considered input, i.e. $T = \frac{\alpha}{\alpha+1} \cdot \frac{a}{b}$. The respective critical point corresponds to the maximum of the marginal productivity and is an inflection point for the dynamics of the output. In fact, the situation when 1<α<2 ensures the fulfilment of the assumptions related to the standard neoclassical production function 1

If α>2, we deal with a third critical point, which is related to the change of the curvature of the marginal productivity, i.e. $T = \frac{\alpha-2}{\alpha+1} \cdot \frac{a}{b}$. In this context, the maximum of the acceleration of the output related to the considered input is obtained. Also, the curvature of the marginal productivity changes. If $T < \frac{\alpha-2}{\alpha+1} \cdot \frac{a}{b}$ the curvature is convex, while if $T > \frac{\alpha-2}{\alpha+1} \cdot \frac{a}{b}$ the curvature is concave.

5. The curvature of the average productivity

Analogously to the marginal productivity we can reveal the curvature of the average productivity of the considered input. Therefore, we have to write the computation formula for the first and second derivative.

The formula of the first derivative of the average productivity ($\frac{d\frac{Y}{T}}{dT}$) is:

$$\frac{d\frac{Y}{T}}{dT} = (\alpha-1) \cdot T^{\alpha-2} \cdot (a - b \cdot T \cdot \frac{\alpha}{\alpha-1}) \qquad (18)$$

We notice that $\frac{d\frac{Y}{T}}{dT} = 0$ if $T = \frac{\alpha-1}{\alpha} \cdot \frac{a}{b}$.

The second derivative of the average productivity ($\frac{d^2\frac{Y}{T}}{d^2T}$) can be expressed as:

$$\frac{d^2\frac{Y}{T}}{d^2T} = (\alpha-1) \cdot (\alpha-2) \cdot T^{\alpha-3} \cdot (a - b \cdot T \cdot \frac{\alpha}{\alpha-2}) \qquad (19)$$

We notice that $\frac{d^2\frac{Y}{T}}{d^2T} = 0$ if $T = \frac{\alpha-2}{\alpha+1} \cdot \frac{a}{b}$

Like in case the marginal productivity, the curvature of the average productivity depends on the size of the parameter α and on the allocated quantity of the considered input.

1D. Debertin (2012) shows that a neoclassical production function assumes that, at the beggings of the usage of an input, the productivity also increases at an increasing rate. The convex curvature of the output growth ends at the inflection point when the maximum of the marginal productivity is obtained. After the inflection point the output experiences a concave curvature as a consequence of the manifestation of the law of the diminishing returns. In this context, the marginal productivity decreases and equal to zero and the maximum level of the output is obtained.

The critical values of the parameter α are the same as in case of the marginal productivity. Hence, we deal with three situations (table 2).

Table 2. The signs of the proposed production function and its derivatives depending on the size of the parameter α and the quantity allocated from the considered input

Allocated quantity of input	$\frac{Y}{T}$	$\frac{d\frac{Y}{T}}{dT}$	$\frac{d^2\frac{Y}{T}}{d^2T}$
0<α<1			
T<((α-2)/α)*(a/b))	>0	<0	>0
((α-2)/α)*(a/b))<T<((α-1)/α)*(a/b))	>0	<0	>0
T>((α-1)/(α)*(a/b))	>0	<0	>0
1<α<2			
T<((α-2)/α)*(a/b))	>0	>0	<0
((α-2)/α)*(a/b))<T<((α-1)/α)*(a/b))	>0	>0	<0
T>((α-1)/(α)*(a/b))	>0	<0	<0
α>2			
T<((α-2)/α)*(a/b))	>0	>0	>0
((α-2)/α)*(a/b))<T<((α-1)/α)*(a/b))	>0	>0	<0
T>((α-1)/(α)*(a/b))	>0	<0	<0

If 0<α<1, we deal with a continuous decreasing average productivity under a convex curvature.

If 1<α<2, we deal with a concave curvature of the average productivity. The maximum of the respective indicator is obtained if $T = \frac{\alpha-1}{\square} \cdot \frac{a}{b}$.

If α>2, we are confronted with a change of the curvature of the average productivity, when $T = \frac{\alpha-2}{\alpha} \cdot \frac{a}{b}$. Hence, the respective curvature is a convex one if $T < \frac{\alpha-2}{\alpha} \cdot \frac{a}{b}$ and is a concave one if $T > \frac{\alpha-2}{\alpha} \cdot \frac{a}{b}$

5. The features of the elasticity of output in the context of the proposed production function

The relationship between the marginal productivity and the average productivity is revealed by the size of the elasticity of the output related to the considered input (EY). Debertin (2012) shows that ratio marginal productivity /average productivity is in fact the elasticity of the output related to the considered input, because we may write:

$$\frac{dY}{dT} \div \frac{Y}{T} = \frac{dY}{Y} \div \frac{dT}{T} = E_Y \quad (20)$$

In the case of production function proposed in this paper, we have:

$$E_Y = (\propto +1) - \frac{a}{a - b \cdot T} \quad (21)$$

We notice that EY decreases as the allocated quantity of the considered input increases. This is in line with the features of a class of neoclassical production functions which assumes that the ratio marginal productivity/average productivity continuously changes depending on the allocated quantity from the considered input (D. Debertin, 2012).

If we consider all the 5 critical points of the production function mentioned before, we obtain the following values of the elasticity of the output:

$E_Y = 0$ if $T = \frac{\alpha}{\alpha+1} \cdot \frac{a}{b}$ (maximization of the output level)

$E_Y = 1$ if $T = \frac{\alpha-1}{\alpha} \cdot \frac{a}{b}$ (maximization of the average productivity)

$E_Y = \frac{\alpha+1}{2}$ if $T = \frac{\alpha-1}{\alpha+1} \cdot \frac{a}{b}$ (maximization of the marginal productivity)

$E_Y = \frac{\alpha+2}{2}$ if $T = \frac{\alpha-2}{\alpha} \cdot \frac{a}{b}$ (point of inflection for the average productivity)

$E_Y = \frac{2(\alpha+1)}{3}$ if $T = \frac{\alpha-2}{\alpha+1} \cdot \frac{a}{b}$ (point of inflection for the marginal productivity)

The results obtained in the cases a) and b) confirm the assumptions of the standard neoclassical theory on the diminishing returns. D. Debertin (2012) shows that when the output is maximized the elasticity of the output related to the considered input is equal to zero. Also, the maximization of the average productivity is obtained in the context of the equality of the marginal productivity and average productivity.

The use of the proposed production function offers the conditions for the estimation of the elasticity of the output in the context of increasing marginal returns. We notice that the elasticity of the output in cases c), d) and e) is dependent on the size of the parameter α. Hence, depending on the values of the above-mentioned parameters the elasticity of the output related to the critical points may sensibly varies.

We remark that the maximization of the marginal productivity is obtained when the elasticity of the output is higher than 1. Also, it is to be mentioned that in the context of a convex curvature for both average and marginal productivity the elasticity of the output is higher than 2.

6. The redefinition of the notion of stages of production

The above-mentioned 5 critical points and the features of the curvatures of the marginal and average productivity allow us to redefine the notion of stages of production. D. Debertin (2012) shows that on the basis of the size of the elasticity of the output three stages of production can be defined, i.e. a) stage I, if the elasticity of the output is higher than 1, b) stage II, if the elasticity of the output varies between 0 and 1 and c) stage III, if the elasticity of the output is negative. The stage II is considered as the realistic economic region, because here the premises for the maximization of the profit occur. The other two cases are treated as irrational ones.

We agree that the stage III can be assumed as an irrational one, but we think that the situations when the elasticity of the output is higher than one needs a more detailed analysis. Therefore, we propose to redefine the notion of stages of production considering the combined curvatures of marginal and average productivity. In this context, we deal with 6 stages of production, which may be grouped into 3 categories of stages of production, respectively:

A.Stages of production defined by the convex curvature of a positive average productivity and an elasticity of the output higher than 2, which include: Stage a, when we deal with a positive marginal productivity and with a convex curvature and Stage b, when we deal with a positive marginal productivity and with a convex curvature.

B.Stages of production defined by the concave curvature of a positive average productivity and an elasticity of the output higher than 1, which include: Stage c, when we deal with a positive marginal productivity and with a concave curvature and Stage d, when we deal with a negative marginal productivity and with a concave curvature.

C.Stages of production defined by the convex curvature of a negative average productivity, which include: Stage e, when we deal with an positive subunit elasticity of the output and Stage f, when we deal with a negative elasticity of the output.

We notice that Stage a, Stage b, Stage c and Stage d are included by D. Debertin (2012) in the Stage I, while Stage e is Stage II and Stage f is Stage III.

The occurrence of the respective stages of production is conditioned by the size of the parameter α. Therefore, if α>2, we deal with all the six stages. If 1< α<2, we deal with Stage c, Stage d, Stage e and Stage f. If α<1, only Stage e and Stage f are met.

7. Conclusion

The production function with one input, which was presented in this paper has important advantages from methodological point of view. Therefore, we are able to deal with non-constant marginal returns. Also, the algebraical properties of the above-mentioned production has the advantage of fulfilment of the most important conditions assumed by the neoclassical theory concerning the correlations of the dynamics of the output and the allocated quantity of considered input, on the one hand, and of extension of the analysis in the situations in case of increasing marginal returns, on the other hand.

The adoption of the assumption of non-constant marginal returns determinates an extension of analysis methodology concerning the feature of the curvature of marginal and average productivity. This way, we can detect the critical points of the dynamics of the productivity. Therefore, it is possible to give an idea concerning the size of elasticity in the context of an accelerated growth of the productivity. Hence, we are able to redefine the notion of stages of production and to focus the attention on the situations when the elasticities of the output are higher than 1.

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