Fuzzy and Deterministic Models in the Simulation of Systems

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***Abstract**: The assertion that uncertainty characterizes the economic processes is already a truism and it is one of their inseparable features.*

The determinism can only have a coordinate of the didactic process in the field of economic sciences.

The deterministic path has been replaced, under the assault of the sciences of complexity, the chaos theory and the fuzzy sets of new approaches, with new openings that make their way and find an increasingly important place in economy.

***Keywords**: system, fuzzy models, simulation, forecast, trapezoidal fuzzy numbers*

***JEL Classification**: C53, D81, E17*

1. Introductory considerations

Perhaps the most dynamic and successful science, physics, has taught us, beyond the beauty of the world and the mysteries of the Universe, that success and achievements (as well as failures) come from much thought, from altruism, from the courage to approach new paths, from the openness to other sciences, from the understanding that within the laws that still seem immutable to us, change and the novelty breathe the progress itself and perhaps one day the good of mankind as well.

Moving into the realm of economic and social realities, we find that during the last three decades our planet has systematically experienced ever-wider and increasingly profound crises, which culminated in the global crisis triggered in 2008, which subjected the society and the economy to a maximum stress.

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It is said that when capitalism is seriously threatened by its own presence, by its own forms of manifestation, a new version of it emerges, a lot better adapted to the existing conditions, especially as the static neoclassical models have proven to be inadequate.

It is neither the place nor the purpose of this paper to analyze the neoliberal dogma or the market fundamentalism and its main corollary, according to which the state intervention in the economy is not beneficial and will not be beneficial.

The transition from linear to nonlinear, from determinism to randomness, from the practice of complication to the science of complexity took by surprise the whole theoretical scaffolding of the economic science and obviously, the theorists and practitioners in the field.

The dialectics between tradition and modernity, between the old and the new, have been and will be permanently in a struggle to ultimately overcome this blocking, perennial dualism which is currently without solutions that do not necessarily involve catastrophes and crisis.

In addition, it would seem that the era of the absurd pragmatism of the separation of sciences in their dynamics and their action has almost ended. Just as in the economic reality where processes and phenomena interfere and they cannot be understood without the understanding of biology, chemistry, physics, mathematics, sociology, and so on, the same goes for other areas of the human activity where these fundamental sciences cooperate and interpenetrate in a transdisciplinary, intradisciplinary and a multidisciplinary approach.

Here as well, the field of economics is far from the other areas of science.

The science of complexity, with its new theories and methods, such as the chaos theory, the catastrophe theory, the fractal theory, the fuzzy techniques, etc. is already present in the most powerful and respectable fields of science: biology, physics, ecology, meteorology, etc.

Without also developing on the subject the issue of using an irreplaceable source of life – water, we cannot help but discuss one of the many aberrations that the classical economy has pushed us and is pushing us to, for example, in the US, given the low price of cereals, on the one hand, and the urban development on the other hand, two phenomena of great magnitude emerged: the "thirst" for water in the urban environment and the abandonment of agricultural activities by farmers. Why? Because the municipalities were buying the water resources on their land at good prices, which caused the farmers to appreciate that, on the one hand, they would have much greater profits just by selling their water resources and, on the other hand, they would have stable profitability guaranteed.

As the whole economic science has profit as its core concept, it is easy to see where such a policy has led and can still lead to in the case of an economic and military superpower. What will happen to other less developed countries if the same reasoning is applied? It's not hard to imagine.

The problem is not simple, but it is solved simplistically, according to the classical and traditional approaches, which can no longer be found among the conceptual tools of the economic science, besides maybe in ever narrower fields. How would Plasma physics, for example, apply to Newton's laws? Similarly, the size and depth of economic problems need a new paradigm, an "escape" to the platoon of "hard-working" sciences, the sciences that already have a tradition (not only) in the science of complexity.

Without actually entering the field of this science we will give an example of ***a comparative analysis between the deterministic approach*** and the fuzzy technique approach in making forecasts in the field of public water supply services.

Beyond the opinions about relativizing the significance of forecasts in an increasingly unstable economic, social and political environment, we remain firmly in the position that asserts and supports the need thereof similarly to a strong light on a route to be traveled at night and in the fog.

Indeed, the very condition of occupying a top management position is based on a series of important indicators whose predicted values depend both on the stability of the managed system, as well as on the entire management team over a long term horizon.

To begin with, let us keep in mind that any mathematical simulation model we adopt will necessarily include three types of variables:

a. variables whose dynamics are systematically nonlinear;

b. variables whose dynamics are quasi-linear;

c. variables whose dynamics are linear (rarer).

These features are valid, obviously, on limited intervals, not for unlimited periods of time, however they are sufficient to cover the forecast horizon (4-5 years) and even longer (5-10 years).

What are the main parameters that must be included in the mathematical model of simulation of physical volume dynamics of water supply and sewerage services?

Any model will include:

- the physical volume of the services (usually noted Q (t), t = time);
- the rate at any given time (T (t));

- the number of employees (NA (t));
- the level of the total operating expenses per 1000 RON revenue, noted with Q1000 (t).

If we show the dynamics of some of these variables using graphs, for example, the number of employees (NA (t)) and the rate (T (t)), perhaps the most common situations from reality would be:

a. Example 1

Figure 1.1

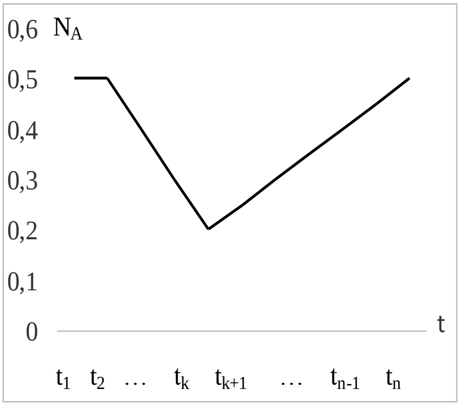


Figure 1.2

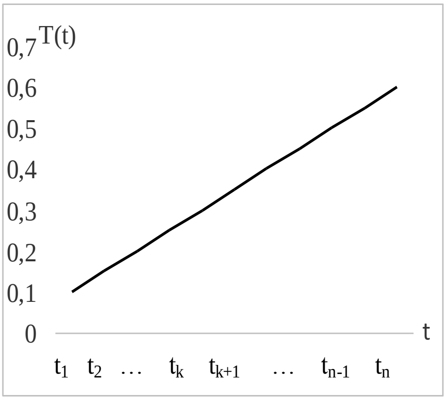


Figure 1.3

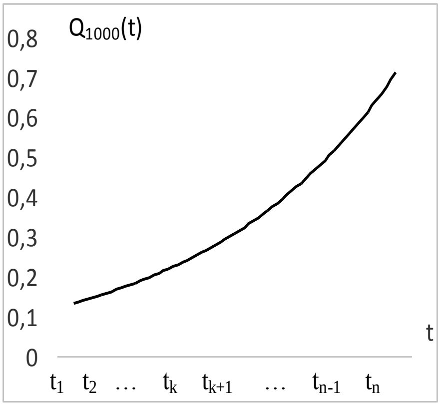
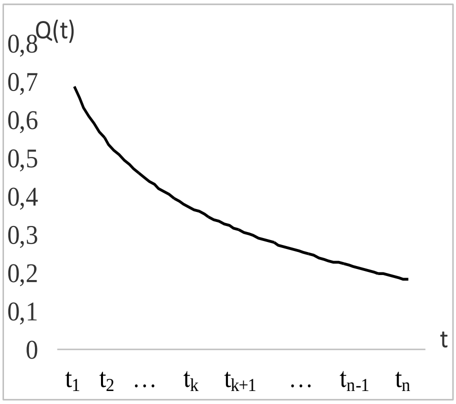


Figure 1.4



b. Example 2:

Figure 1.5

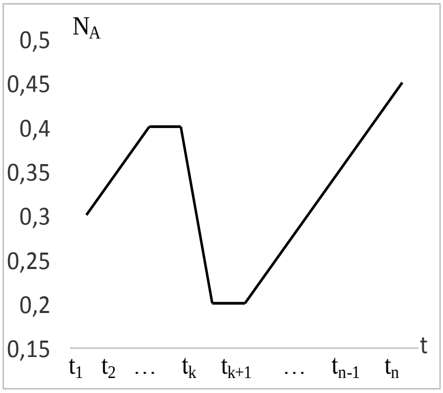
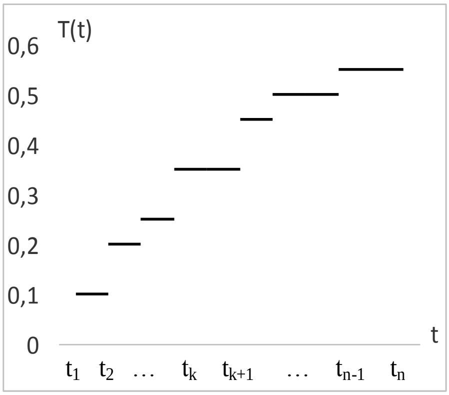


Figure 1.6



In the case of ***the first example***, in terms of the analyzed parameters, the following brief explanations can be brought:

a. The number of employees, NA(t) has been constant for a considerable period of time after which a large-scale restructuring followed (let us suppose that 20% of staff was restructured); thereafter, the number of employees increased linearly and at a rather low, "cautious" growth rate.
b. The rate has had a linear increase throughout the time frame.
c. The physical volume of services had an asymptomatic downward trend towards a physical volume noted by Q0, whereas,
d. The level of expenses per 1000 RON revenue (or turnover) experiences a non-linear upward path (after a very "quiet" exponential).

For ***the second example***, only the number of employees NA (t) and the rate T (t) were taken into consideration.

a. After a period of relatively accelerated growth, a short period follows in which the number of employees remains capped, followed by a strong reduction in a relatively short time followed again by a capping period, after which the number of staff increases linearly and at a relatively slow pace.
b. The rates experience a "scale" type of increase over the whole time horizon, without being compulsory for the specific times of each rate to be equal to each other (unequal "steps").

These examples were given for the following reasons:

a. Rarely can theoretical curves be found in practice, in the reality of the functioning of socio-productive systems;

b. The linear increase of the rate, for example (the graph shown in fig.1.2), in practice can never have such a trajectory (the very rate increase from second to second, from minute to minute, years in a row, is not possible in reality). But surely, the "scale function" type of increase exists.

c. The dynamics for the physical volume of services (Q (t)) and the level of expenditure for an income of 1000 RON (Q1000 (t)) are somewhat idealized. In the very long term (10, 15, 20 years) such dynamics can be found, but certain (increasing or decreasing) deviations from the trajectory presented graphically may appear (and even exist) within them.

For example, one can very frequently encounter situations such as:

a. For the last 25 years, the dynamics of the physical volume of service has been that in Figure 1.7.

Figure 1.7

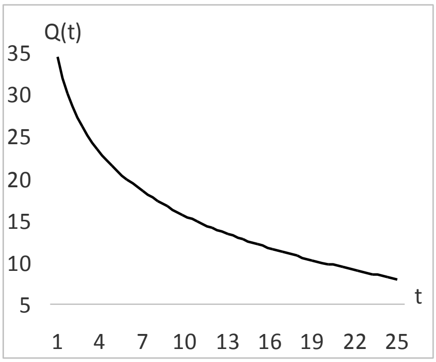
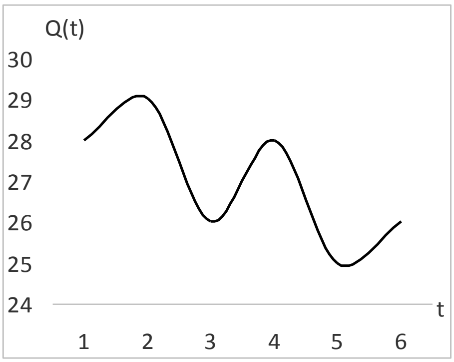


Figure 1.8



b. If we "extract" a period of 5-6 years from this time horizon, then we might encounter a dynamic such as the one shown in Figure 1.8.

For strategic analysis and for defining policies in the field of public water supply services, very long-term data series are predominantly used in the very long term

(Figure 1.7.) but for defining a management plan, the dynamics represented in Figure 1.8. becomes the reference.

2. The general expression of the deterministic model of forecasting

We do not claim to develop a unique and universally valid model, but only a subclass of models, starting on the one hand from the economic realities and on the other hand, from the desire to capture what is essential in the functioning of the systems dedicated to the public water supply and sewerage services, without exponentially and unnecessarily complicating the simulation models by using sophisticated mathematical tools. Therefore, let us consider the time horizon [t0, tn] so that:

System A1

$$N_A(t) = \{ f_i(t) = k_j \} \quad , i = \overline{1,p}; \; j = \overline{1,m} \quad \text{(scale function)} \tag{1}$$

$$T(t) = \{ g_l(t) = b_r \} \quad , l = \overline{1,s}; \; r = \overline{1,n} \quad \text{(scale function)} \tag{2}$$

$$Q_f(t) = a + \frac{b}{t} + \frac{c}{t^2} \quad , \text{where } a, b, c = \text{constants to be defined} \tag{3}$$

$$Q_{1000}(t) = \alpha \cdot N_A(t) + \beta \quad , \text{where } \alpha, \beta = \text{constants to be determined} \tag{4}$$

$$\mathrm{Q_V}(t) = T(t) \cdot Q_f(t) \quad , \mathrm{Q_V}(t) = \text{the value of the physical volume of services [RON]} \tag{5}$$

$$C_T(t) = Q_V(t) \cdot Q_{1000}(t) \quad , \mathrm{C_T}(t) = \text{total operating expenses [RON]} \tag{6}$$

$$\mathrm{P_r}(t) = Q_V(t) - C_T(t) \quad , \mathrm{P_r}(t) = \text{gross profit} \tag{7}$$

Specifically, if we consider a time horizon of 10 years [t0, t9], then the system A1 can take the following form:

System B1

$$\begin{cases} N_A = \begin{cases} f_1(t) = 2500 & , t_0 \le t \le t_3 \\ f_2(t) = 1800 & , t = t_4 \\ f_3(t) = f_2(t-1) + 100 & , t_5 \le t \le t_9 \end{cases} \\ T = \begin{cases} g_1(t) = 3{,}57 & , t_0 \le t \le t_4 \\ g_2(t) = 4{,}10 & , t_5 \le t \le t_9 \end{cases} \\ Q_f(t) = 40000 + \dfrac{10000}{t} + \dfrac{3000}{t^2} \quad , \forall t \in [t_0, t_9] \\ Q_{1000}(t) = \dfrac{0.3 \cdot N_A(t) + 200}{1000} \\ Q_V(t) = T(t) \cdot Q_f(t) \qquad , \forall t \in [t_0, t_9] \\ C_T(t) = Q_V(t) \cdot Q_{1000}(t) \\ P_r(t) = Q_V(t) - C_T(t) \end{cases} \tag{8}$$

In system B1, the rate has a two-step "scale" function expression but it can also have a linear function expression (for example: $T = 0.006 \cdot t + 3.44$).

The data can be organized in a table of the following form:

Table 1. Data source

Time sequences	NA(t)	T(t)	Qf(t)	Q1000(t)	QV(t)	CT(t)	Pr(t)
t0= 1	2500	3.57	53000	0.95	189210	179750	9461
t1= 2	2500	3.57	45750	0.95	163328	155162	8166
t2= 3	2500	3.57	43667	0.95	155891	148096	7795
t3= 4	2500	3.57	42688	0.95	152396	144776	7620
t4= 5	1800	3.57	42120	0.74	150368	111272	39096
t5= 6	1900	4.10	41750	0.77	171175	131805	39370
t6= 7	2000	4.10	41490	0.80	170109	136087	34022
t7= 8	2100	4.10	41297	0.83	169318	140534	28784
t8= 9	2200	4.10	41148	0.86	168707	145088	23619
t9=10	2300	4.10	41030	0.89	168223	149718	18505

Source: data processed by the authors

One can observe that the "leading functions" are those that describe the trajectories for:

- the number of employees: NA(t);
- the average rate: T(t);
- physical volume of service: Qf(t).

For the system B1 we obtain the following graphical representations for the three leading relationships:

Figure 2.1

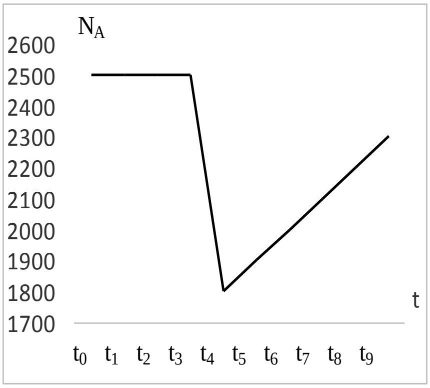


Figure2.2

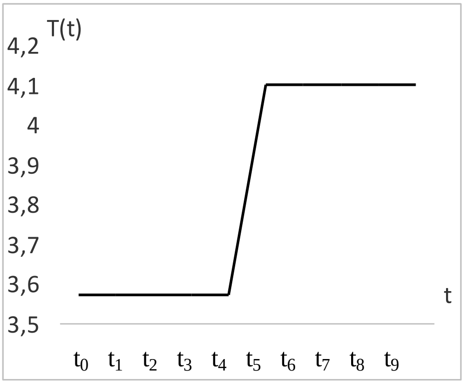
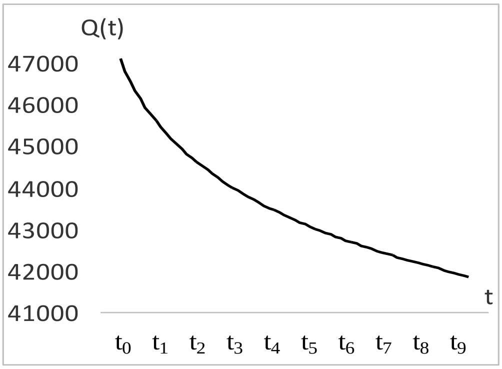


Figure2.3



It should be emphasized that if, in the class of type A models, the "pivot" parameters are the number of employees, the rate and the physical volume of the services, one can imagine models in which the "pivot" parameters are represented by the consumption of electricity, the level of reimbursements or the outsourcing, if appropriate.

3. Forecasts through fuzzy techniques

The deterministic trajectories, resulting from the simulation models presented briefly in the previous paragraph, have a dominant methodological and didactic utility because predictions of this type have a lower probability of being found in an increasingly unstable and "tormented" economic, social and political environment.

The deterministic "line" or "path" in the graphs above should therefore be replaced by the "lanes" of possible evolutionary variants, which can be defined by fuzzy techniques. The known horizon and the forecast horizon are as follows:

$$t \in \left\{ \underbrace{t_0, t_1, \dots, t_9}_{\substack{known \\ horizon}}, \underbrace{t_{10}, t_{11}, t_{17}}_{\substack{forecast \\ horizon}} \right\} = \left\{ \underbrace{1, 2, \dots, 10}_{\substack{known \\ horizon}}, \underbrace{11, 12, \dots, 18}_{\substack{forecast \\ horizon}} \right\} = \{\underline{2008, 2009, \dots, 2017}, \underline{2018, 2019, \dots, 2025}\} \quad (9)$$

In this paragraph we will perform a fuzzification of the presented model. The uncertainty of the input data (NA, T, Qf as well as the constants of system relationships), can be modeled using trapezoidal fuzzy numbers (with 4 real, ordered components).

Thus, we will consider that ***the number of employees*** for the forecast horizon [t10, t17] ***is the trapezoidal fuzzy number***:

$$\tilde{N}_A = (n_1, n_2, n_3, n_4) \ , \ n_1 \le n_2 \le n_3 \le n_4 \quad (10)$$

This is interpreted as follows: the projected number of employees will be between the n2 and n3 values.

However, in special cases, the number may drop to n1 (mass layoffs) or climb to n4 (e.g. in the case of substantial subsidies, when obtaining European grants, etc.). However, it is assumed that the number of employees will not exceed these final limits (below n1 or above n4).

Similarly, ***the forecasted rate for drinking water*** will be modeled with trapezoidal fuzzy numbers.

The parameters in his expression of Q_f are also modeled by fuzzy numbers (fuzzy constants):

$$\tilde{a} = (38500, 40000, 40500, 41000)$$

$$\tilde{b} = (8400, 9900, 10600, 11100)$$

$$\tilde{c} = (2800, 2900, 3100, 3200)$$

Finally, the fuzzy system is as follows:

$$\text{System F} \begin{cases} \tilde{N}_A(t) = (1800, 1850, 2100, 2250) \cdot Fd_1(t) \\ \tilde{T}(t) = (3.8, 4.4, 5.0, 5.2) \cdot Fd_2(t) \\ \tilde{Q}_f(t) = \left(\tilde{a} + \frac{1}{t} \cdot \tilde{b} + \frac{1}{t^2} \cdot \tilde{c} \right) \cdot Fd_2(t) \\ \tilde{Q}_{1000}(t) = 0.0002 \cdot \tilde{N}_A(t) + (0.25, 0.28, 0.32, 0.35) \\ \tilde{\mathrm{Q}}_{\mathrm{V}}(t) = \tilde{T}(t) \cdot \tilde{Q}_f(t) \cdot \mathrm{Fd}_3(t) \\ \tilde{C}_T(t) = \tilde{Q}_V(t) \cdot \tilde{Q}_{1000}(t) \cdot Fd_4(t) \\ \tilde{\mathrm{P}}_{\mathrm{r}}(t) = \tilde{Q}_V(t) - \tilde{C}_T(t) \end{cases} , \quad (\forall)\, t > t_9 = 10 \quad (t \in [11, 18]) \tag{11}$$

whereFdk are dispersion factors of 3% - 5% annually:

$$Fd_1(t) = 1 + (t - 10) \cdot 0.05$$

$$Fd_2(t) = 1 + (t - 10) \cdot 0.03$$

$$Fd_3(t) = 0.93 - (t - 10) \cdot 0.03$$

$$Fd_4(t) = 1.13 - (t - 10) \cdot 0.03$$

In the article [Zadeh L.A.(1965)], the introductory notions of fuzzy theory were defined for the first time.

To this day, half a century later, the fuzzy theory has made substantial contributions to many fields of knowledge.

Amongst the many works of theory and fuzzy applications that have appeared, we mention here only a few in which various definitions for fuzzy number operations can be consulted: Kaufmann (1973), Kaufmann&Aluja (1995), Bojadziev(1995), Tacu(1998), Gherasim (2008), Vlădeanu (2008) and Alecu (2012).

In the article we will use for the operations with trapezoidal fuzzy numbers the definitions in the Gherasim (2004) paper.

Let us consider two trapezoidal fuzzy numbers:

$\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$,cu $a_1 \le a_2 \le a_3 \le a_4$ and $b_1 \le b_2 \le b_3 \le b_4$.

The actual associated number *called* ***the center of gravity*** *is the arithmetic mean:*

$$a_G = <\tilde{a}> = \frac{a_1 + a_2 + a_3 + a_4}{4} \tag{12}$$

Multiplying a fuzzy number with a scalar *(real number) is done by each component, and if the scalar is negative, the order of the components is reversed:*

$$t\tilde{a} = \begin{cases} (ta_1,\ ta_2,\ ta_3,\ ta_4) & ,t > 0 \\ (ta_4,\ ta_3,\ ta_2,\ ta_1) & ,t < 0 \end{cases} \quad t \in R \tag{13}$$

The addition of two fuzzy numbers *is performed with each component, and for subtraction the components of the subtractor are reversed:*

$$\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \tag{14}$$

$$\tilde{a} - \tilde{b} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1) \tag{15}$$

Multiplying and dividing fuzzy numbers *is done only with the scalar multiplication and addition operations (already defined):*

$$\tilde{a} \cdot \tilde{b} = \frac{\tilde{a} \cdot b_G + a_G \cdot \tilde{b}}{2} \tag{16}$$

$$\frac{\tilde{a}}{\tilde{b}} = \frac{\tilde{a} \cdot b_G + a_G \cdot \tilde{b}}{2b_G^2} \tag{17}$$

Let us calculate, for example $\tilde{N}_A(t_{10})$ where $t_{10} = 11$:

$$Fd_1(11) = 1 + (11 - 10) \cdot 0.05 = 1.05$$

$$\tilde{N}_A(11) = (1800, 1850, 2100, 2250) \cdot 1.05$$
$$= (1.05 \cdot 1800, 1.05 \cdot 1850, 1.05 \cdot 2100, 1.05 \cdot 2250)$$

$$\tilde{N}_A(11) \approx (1890, 1943, 2205, 2363)_{2100}$$

The index 2100 is the center of gravity associated with the fuzzy number.

The trapezoidal fuzzy numbers representing the indicator $\tilde{N}_A(t),\ t = \overline{12,18}$ for the entire forecast horizon, are calculated similarly.

Let us also calculate $\tilde{T}(11)$: $Fd_2(11) = 1 + (11-10)\cdot 0.03 = 1.03$

$\tilde{T}(11) = (3.8, 4.4, 5.0, 5.2)\cdot 1.03 \approx (3.9, 4.5, 5.2, 5.4)_{4.7}$

The other rates are obtained in a similar fashion.

Calculating the (fuzzy) indicator $\tilde{Q}_f(11)$ is a bit more complex:

$$\tilde{Q}_f(11) = \left(\tilde{a} + \frac{1}{11}\cdot\tilde{b} + \frac{1}{11^2}\cdot\tilde{c}\right)\cdot Fd_2(11) =$$

$$= (38500,40000,40500,41000) + \frac{1}{11}\cdot(8400,9900,10600,11100)$$

$$+\frac{1}{121}\cdot(2800,2900,3100,3200) =$$

$$= (40465,42152,42734,43297)_{42162}$$

The other $\tilde{Q}_f$ indicators for the forecast horizon are calculated smiliarly.

$$\tilde{Q}_{1000}(t) = 0.0002\cdot\tilde{N}_A(t) + (0.25, 0.28, 0.32, 0.35) \quad , \forall\, t > 11$$

For the $\tilde{Q}_{1000}$ ***fuzzyindicator*** the calculations go as follows:

$$\tilde{Q}_{1000}(11) = 0.0002\cdot\tilde{N}_A(11) + (0.25, 0.28, 0.32, 0.35) =$$

$$\approx (0.38, 0.39, 0.44, 0.47) + (0.25, 0.28, 0.32, 0.35)$$

$$\approx (0.63, 0.67, 0.76, 0.82)_{0.72}$$

For the $\tilde{Q}_V$ ***fuzzyindicator*** we have the following calculation:

$$Fd_3(11) = 0.93 - (11-10)\cdot 0.03 = 0.90$$

$$\tilde{T}(11)\cdot\tilde{Q}_f(11) = (3.91, 4.53, 5.15, 5.36)_{4.7375}\cdot(40465,42152,42734,43297)_{42162} =$$

$$= \frac{42162\cdot(3.91, 4.53, 5.15, 5.36) + 4.7375\cdot(40465,42152,42734,43297)}{2} \approx$$

$$\approx \frac{(164853,190994,217134,225988) + (191703,199695,202452,205120)}{2} \approx$$

$$\approx (178278, 195345, 209793, 215554)$$

$$\tilde{Q}_V(11) = \tilde{T}(11) \cdot \tilde{Q}_f(11) \cdot Fd_3(11) = (178278, 195345, 209793, 215554) \cdot 0.9 \approx$$

$$\approx (160450, 175810, 188814, 193999)_{179768}$$

For the other $\tilde{Q}_V$ indicators and for the $\tilde{\mathbf{C}}_{\mathbf{T}}$, $\tilde{\mathbf{P}}_{\mathbf{r}}$ indicators the fuzzy trapezoidal number products are calculated similarly.

After the performance of all these calculations, the results were centralized in the table presented in Annex 1.

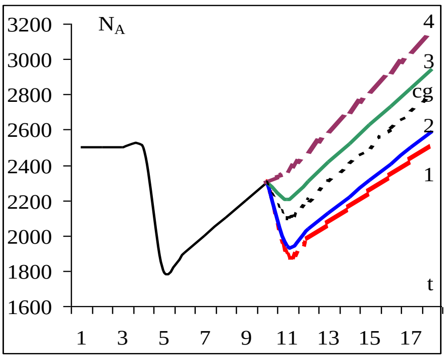
For example, it can be noted from the mentioned Annex that:

$$\tilde{N}_A(11) = (1890, 1943, 2205, 2363)_{2100}.$$

The index 2100 represents the center of gravity (cg = arithmetic mean of the first four components) corresponding to the fuzzy number.

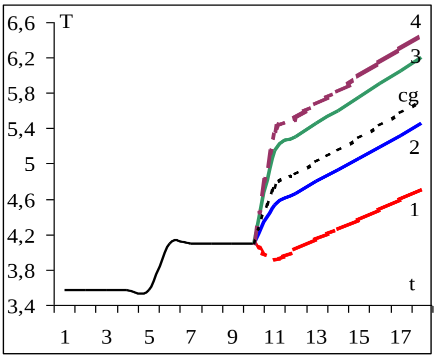
In the following graphs, the four components of the fuzzy trapezoidal numbers are numbered 1, 2, 3 and 4 and the center of gravity is marked "cg".

Figure 3.1. Graphical representation of $\tilde{\mathbf{N}}_{\mathbf{A}}$ indicator – Number of employees



Known horizon | Forecast horizon

Figure 3.2. Graphical representation of $\tilde{\mathbf{T}}$ indicator - unitary rate for 1 m3 of water



Known horizon | Forecast Horizon

Figure 3.3. Graphical representation of $\tilde{Q}_f$ indicator - Physical volume of water supply services

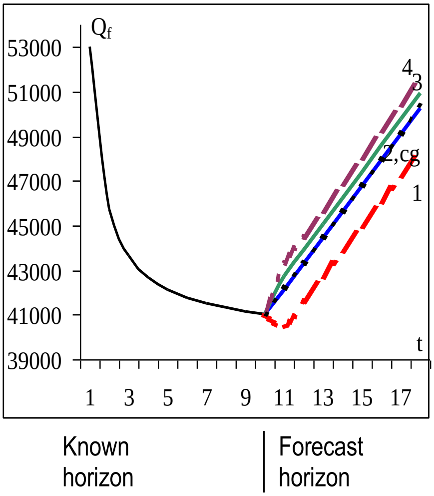
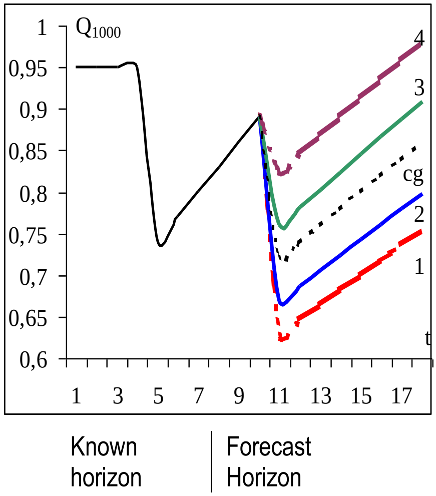


Figure 3.4. Graphical representation of $\tilde{Q}_{1000}$ indicator - operational expenses amounting to 1000 RON revenue



It should be mentioned mention that for the $\tilde{Q}_f$ physical volume of the services, the center of gravity is very little different from the 2nd component (it differs by about one thousand). Graphically, the two curves seem superimposed.

Figure 3.5. Graphical representation of $\tilde{Q}_V$ indicator - Income

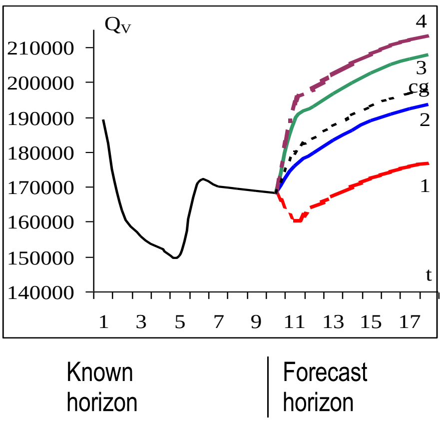


Figure 3.4. Graphical representation of $\tilde{C}_T$ indicator – Total expenses

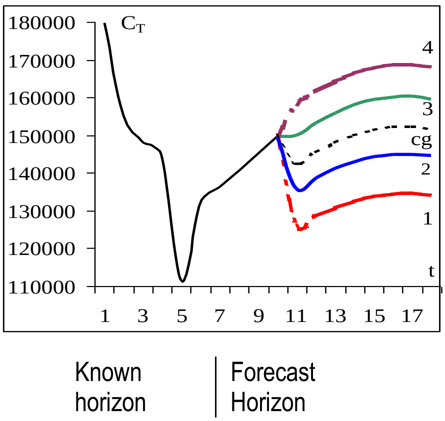
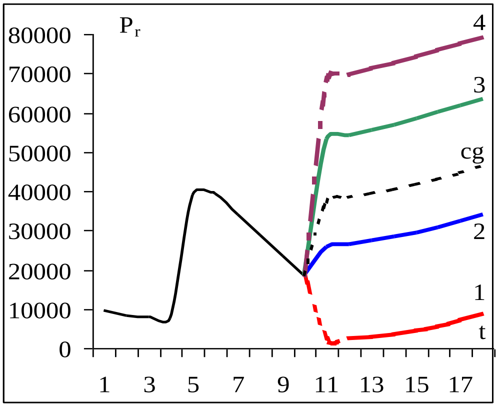


Figure 3.7. Graphical representation of $\tilde{P}_r$ indicator – Gross profit



Known horizon | Forecast horizon

4. Conclusions

The transition from the "pathway" indicated by the deterministic forecasts to the "lane" obtained by the fuzzy techniques determines us to ask a question whose answer, according to the authors, deserves sustained scientific debates.

Therefore, would not the development of a science imply a broadening of its definition and a redefinition of its aims, methodologies and field of research? We believe it would.

The science of complexity shows to us that a complex system is made up of structural and structured components that have optimal meaning and functionality only in the integrity of the system and, except for a short interval of its life, it is generally unpredictable in a deterministic sense.

Also, it shows the existence of the possibility of sudden transformations of great amplitude of the output vectors of the system, due to small, insignificant deviations of an internal or external variable (therefore from the space of the input vectors or of transformation and/or transfer vectors).

At the same time, the complex system differs from the so-called "complicated" system not by the fact that analysts or researchers have difficulty identifying all the variables that would influence the system but by the sensitivity to changes of the initial conditions (from slightly different initial conditions, strongly divergent evolution trajectories may result).

In other words, the dynamics and evolution of a complex system are two different, distinct problems (the evolution does not always result from the dynamics of the response to a given system).

Designed in the area of science and economic realities, these highlights can be transposed and translated into analyzes on complex issues whose succinct formulation could be:

a. Which is most suitable for the social-economic and ecological balance: production systems that operate on the criterion of minimizing the expenses or on the criterion of maximizing profit? (the two criteria are obviously not equivalent).

b. Is efficiency a necessary but not sufficient condition for a system to be economical?

The Science of Complexity is a new way of interdisciplinary approach to reality and should not be confused with a number of models, theories, and/or techniques that they use.

The Science of Complexity studies, par excellence, the field of systems far from equilibrium, and we hardly can find more such examples in the different fields of study compared to the economic and social field.

Also, in an era of globalization, tens and hundreds of thousands of economic agents and businesses (from small to corporate dinosaurs) emerge and disappear. Again, it should be recalled that the Science of Complexity studies not existing systems but the genesis processes thereof (emergence, self-organization) and their mode of operation under the action of fluctuations (both those intrinsically existing in the environment and those generated by the actions of the system itself).

The fundamental condition and the fundamental criterion of functioning of the socio-productive (or socio-economic) systems are concepts whose status usually stops at the level of simplistic and dual indicators such as productivity, GDP, profit, etc.

It is obvious that as far as possible but without triumphalist exaggerations, the mathematical approach remains one of the most powerful techniques of penetrating the fog and/or darkness of less thorough research or of less accessible fields.

The mathematical models must and can represent the realities of the economic and social life in what is defining, essential, perennial.

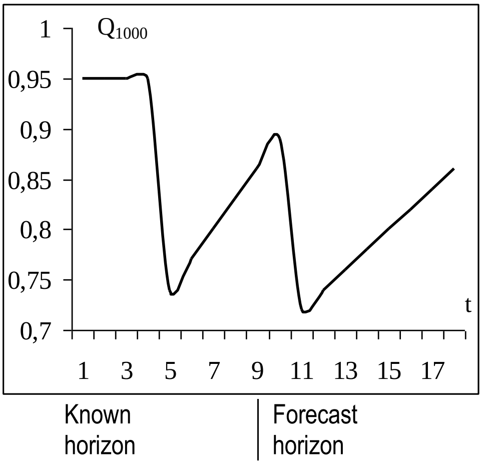
The mathematical models and modeling are a very good working tool but they do not solve all the problems and they cannot fulfill all the requirements and/or restrictions.

One of the conditions that a mathematical model must meet is to respect as best as possible the trend, the long-term behavior of the modeled indicator or process.

For example, for the case study considered, if the expenses per 1000 RON revenue (Q1000) indicator has a behavior of a certain type in the 10 years that define the known (or past) horizon, then, logically, there is no argument for this behavior to change in the forecast horizon.

If for the forecast horizon we would use only the value of the center of gravity (to simplify the analysis), we would obtain the following graph:

Figure A.1 Graphical representation of $\mathbf{Q_{1000}}$ ***Indicator - Operational expenses per 1000 RON revenue***



It can be easily observed that "past allure" of the curve is preserved according to the system-specific principle, which states that any system memorizes its state.

This (very important) aspect can also be seen in the case of the indicators $\tilde{\mathbf{Q}}_{\mathbf{V}}$ and $\tilde{\mathbf{C}}_{\mathbf{T}}$.

It should also be emphasized that the "fuzzy mathematics" (better known as "fuzzy arithmetic") is not the same as the deterministic mathematics in the sense that while in the latter case there is the equality: income - expenses = profit ($Q_V - C_T = P_r$), in the field of fuzzy mathematics this equality is no longer verified (except at the level of the values of the centers of gravity).

Let us also emphasize that the parameters (indicators) considered were "allowed" to evolve "freely", with no additional relationships of interconditioning used or imposed amongst them (such as for example that the profit should not exceed 15% of the revenues).

The introduction of such conditions or restrictions is not a problem in itself and remains at the discretion of the analyst, researcher or economist whether or not he or she uses such restrictions.

Finally, the utility for any factor or group that decides to have information on the spectrum of possible values to be browsed in any forecast horizon by the important indicators that characterize the dynamics of the analyzed system should be emphasized.

The ideas outlined in this article intended, using a relatively simple example, to once again highlight the need to intensify efforts along the lines of approaches further away from the old ideas and ideologies and closer to the perceptions of the Science of Complexity, which is also one of the ways by which the concerns in the economic field of a whole body of specialists, researchers, teachers to raise the prestige of economic science, quite often disadvantaged during the last decades.

The article presented only a small number of the concepts of the Science of Complexity for which the specialized literature provides an increasing range of references, of which we mention: Nassim (2008), Munteanu (2008), Prigogine (1984), Kaye (1999), Vlădeanu (2008).

The resilience of the economic systems, the inseparability, unpredictability and behavior far from equilibrium, the unrepeatability of the past, the non-linearity, the rarity of the anthropic factors and the increasing density of the entropic factors, the transition of the systems from the operation in the field of time to that of frequency, etc. are problems that have to concern in a more comprehensive and dynamic way the groups of researchers in the field of economic science because it would seem that the rarest resource will be neither water nor oil but time.

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Annex 1

	t	Fd1	NA(t)	Fd2	T(t)	Qf(t)	Q10	Fd3	QV(t)	Fd4	CT(t)	Pr(t)
Known horizon	1		2500		3.57	53000	0.950		189210		179750	9461
	2		2500		3.57	45750	0.950		163328		155162	8166
	3		2500		3.57	43667	0.950		155891		148096	7795
	4		2500		3.57	42688	0.950		152396		144776	7620
	5		1800		3.57	42120	0.740		150368		111272	39096
	6		1900		4.10	41750	0.770		171175		131805	39370
	7		2000		4.10	41490	0.800		170109		136087	34022
	8		2100		4.10	41297	0.830		169318		140534	28784
	9		2200		4.10	41148	0.860		168707		145088	23619
	10		2300		4.10	41030	0.890		168223		149718	18505
Forecast horizon	11	1.05	1890	1.03	3.91	40465	0.628	0.90	160450	1.10	125635	2289
		1.05	1943	1.03	4.53	42152	0.669	0.90	175810	1.10	135732	25793
		1.05	2205	1.03	5.15	42734	0.761	0.90	188814	1.10	150018	53082
		1.05	2363	1.03	5.36	43297	0.823	0.90	193999	1.10	158161	68364
		1.05	2100	1.03	4.74	42162	0.720	0.90	179768	1.10	142386	37382
	12	1.10	1980	1.06	4.03	41573	0.646	0.87	164077	1.07	128436	2622
		1.10	2035	1.06	4.66	43296	0.687	0.87	179599	1.07	138610	26383
		1.10	2310	1.06	5.30	43889	0.782	0.87	192913	1.07	153216	54303
		1.10	2475	1.06	5.51	44464	0.845	0.87	198088	1.07	161456	69652
		1.10	2200	1.06	4.88	43306	0.740	0.87	183669	1.07	145429	38240
	13	1.15	2070	1.09	4.14	42687	0.664	0.84	167215	1.04	130753	2884
		1.15	2128	1.09	4.80	44449	0.706	0.84	183250	1.04	141141	27322
		1.15	2415	1.09	5.45	45054	0.803	0.84	196661	1.04	155927	55520
		1.15	2588	1.09	5.67	45641	0.868	0.84	202005	1.04	164331	71253
		1.15	2300	1.09	5.02	44458	0.760	0.84	187283	1.04	148038	39245
	14	1.20	2160	1.12	4.26	43808	0.682	0.81	170120	1.01	132581	3708
		1.20	2220	1.12	4.93	45609	0.724	0.81	186257	1.01	142975	28284
		1.20	2520	1.12	5.60	46226	0.824	0.81	199923	1.01	157972	56948
		1.20	2700	1.12	5.82	46826	0.890	0.81	205239	1.01	166412	72658
		1.20	2400	1.12	5.15	45617	0.780	0.81	190385	1.01	149985	40400
	15	1.25	2250	1.15	4.37	44933	0.700	0.78	172432	0.98	133807	4506
		1.25	2313	1.15	5.06	46774	0.743	0.78	188819	0.98	144261	29428
		1.25	2625	1.15	5.75	47404	0.845	0.78	202708	0.98	159391	58447
		1.25	2813	1.15	5.98	48017	0.913	0.78	208169	0.98	167926	74361
		1.25	2500	1.15	5.29	46782	0.800	0.78	193032	0.98	151346	41685
	16	1.30	2340	1.18	4.48	46062	0.718	0.75	174308	0.95	134463	5602
		1.30	2405	1.18	5.19	47943	0.761	0.75	190904	0.95	144913	30773
		1.30	2730	1.18	5.90	48586	0.866	0.75	204979	0.95	160131	60066
		1.30	2925	1.18	6.14	49213	0.935	0.75	210571	0.95	168706	76108
		1.30	2600	1.18	5.43	47951	0.820	0.75	195191	0.95	152053	43137
	17	1.35	2430	1.21	4.60	47195	0.736	0.72	175900	0.92	134611	7185
		1.35	2498	1.21	5.32	49117	0.780	0.72	192484	0.92	144967	32297
		1.35	2835	1.21	6.05	49772	0.887	0.72	206706	0.92	160187	61739
		1.35	3038	1.21	6.29	50413	0.958	0.72	212234	0.92	168716	77623
		1.35	2700	1.21	5.57	49124	0.840	0.72	196831	0.92	152120	44711
	18	1.40	2520	1.24	4.71	48329	0.754	0.69	176858	0.89	134120	8792
		1.40	2590	1.24	5.46	50293	0.798	0.69	193739	0.89	144457	34171
		1.40	2940	1.24	6.20	50962	0.908	0.69	207897	0.89	159568	63440
		1.40	3150	1.24	6.45	51617	0.980	0.69	213525	0.89	168066	79405
		1.40	2800	1.24	5.71	50300	0.860	0.69	198004	0.89	151553	46452

Within the forecast horizon area each cell contains 5 components.

The first 4 components represent the trapezoidal fuzzy number corresponding to the indicator in the respective column, and the 5th component is the center of gravity (cg = the arithmetic mean of the first 4 components) corresponding to the fuzzy number