

BACK-TESTING THE EFFECTIVENESS OF VALUE AT RISK MODEL

Sanel HALILBEGOVIC¹, Nedim CELEBIC²,
Adisa ARAPOVIC³, Mia VEHABOVIC⁴

Abstract: Value at risk (VaR) is the biggest loss of portfolio that can be expected in the reporting period, with a given level of confidence. Research aims to estimate the effectiveness and accuracy of VaR models, while estimating which test used for back-testing is most reliable in evaluating the VaR model accuracy. Various back-testing methods are used for examining exception frequency and results indicate that the VaR models used are accurate at almost all levels of confidence with only a small possibility of risks and problems. Five tests including Point of Failure, Time until First Failure, Basel Traffic Light, Christoffersen's Independence test and Mixed Kupiec test will be performed to evaluate if the respective method for VaR calculation is consistent. Limitations in the back-testing process are linked to the fact that VaR models are accurate only under normal market conditions.

Keywords: Value at risk, Back-testing, Kupiec, Christoffersen, Basel Traffic Light

JEL Classification: D81; G11; G32

Introduction

We often find ourselves in a position to take a decision without knowing all the consequences and the uncertainties that can be brought by that decision, and furthermore, some of the consequences may be unfavourable. There isn't any accurate definition for risk, but the common elements of all definitions are uncertainty and loss. The risk means any uncertain situation in the business and the probability of loss (gain

¹ Assoc. Prof. Dr.; International Burch University; sanel.halilbegovic@ibu.edu.ba.

² Assist. Prof. Dr.; International Burch University; nedim.celebic@ibu.edu.ba

³ Assoc. Prof. ; International Burch University ; Dr. adisa.arapovic@ibu.edu.ba

⁴ Research fellow ms; International Burch University

reduction) as a result of uncertain events in the business. The most famous type of risk that is related to the securities is market risk, which relates to the uncertainty regarding the change in the price of securities (Halilbegovic, 2016). Risk management has become an integral part of financial operations. This is a process by which to identify, measure and control the exposure to risk (Graham & Pal, 2014). The main objective of risk management is to optimize the relationship between risk and profit. In their business, financial institutions are nowadays faced with two major challenges: risk management and profit maximization. This represents a difficult task, since the risks are numerous and difficult to be identified and even more difficult to be controlled. Financial institutions increase revenue by risk-taking and managing it. Therefore, for the profitability of the institution, the relationship between the risk and income management is of crucial importance (Emmer, Kratz and Tasche 2015). The risk can be estimated using various measures. The first ideas for assessing portfolio risk came from Markowitz, who measured the risk using mean variance behaviour (Markowitz, 1991) . Two measures of risk later emerged: VaR (Value at Risk) and CVaR (Conditional Value at Risk). VaR has become the main measure of risk in banking regulations and in internal risk management of banks (Pflug 2000). VaR is much easier to calculate than most measures for risk and therefore takes an important position in practice. During 1996, 99% VaR is accepted by the Basel Accord as the main measure of risk for determination of possible loss.

1. Literature review

Value at risk, known as VaR, is a measure of risk investment in the financial market. It represents the greatest loss that can be expected in a given interval of time, with a given level of confidence. It is important to note that the VaR is only an estimation of the possible loss. One of the advantages of VaR is that it is a simple, easily understandable number, a measure of risk which the institution is exposed to within the financial market. VaR has its roots in Markowitz's theory of portfolio. Specifically, the methodology underlying the VaR is a result of integrating modern portfolio theory and statistical analysis, which examines the risk factors (Zhang, Nadarajah 2017). In 1998, banks began to use VaR as a measure of risk for calculating the necessary regulatory funds. VaR was introduced by Dennis Weatherstone, chairman of the US bank JP Morgan, with the aim to give him the opportunity to control the daily risk exposure of his company. He gave the task to his analysts to submit a report to him every day - which will be just a number that indicates the potential loss of the day (Campbell, 2005). The participation of positions during the observed period in the portfolio is fixed, which means that VaR provides an opportunity to assess the potential loss (if the structure of the portfolio is not changed) (Yamai 2002). Since it is a value that is calculated with a certain level of confidence, about the estimated losses is possible to speak only as of

the *potential*, and it cannot be said that this is a number which indicates the maximum extent of feasible and safe loss. Thus, VaR does not indicate potential loss in extreme market conditions. For example, if the level of confidence is 95%, the calculated indicator says that it should not be lost more than the stated amount in 95%; but does not tell what might happen in the remaining 5% of cases. According to Jorion (Jorion 2001) the formula for VaR is expressed as:

$$VaR = a * \sigma * W \quad (1)$$

Variables in formula (1) are a – confidence interval, σ – Standard deviation (volatility) and W – Initial portfolio value. VaR takes into consideration how changes in prices of financial instruments affect each other. Therefore, it can reduce the risk with the help of diversification (Christofferssen 1998). Thus, VaR is a measure of risk in the portfolio for the usual business, to a given level of confidence. Therefore, VaR is not efficient in terms of the extreme changes in the market and therefore it should be combined with stress tests, in order to obtain a wider framework for the observation of market risk (Bams, Blanchard, Lehnert 2017). Before doing any calculations and drawing conclusions, it is important to be aware of all crucial terms regarding the process of forecasted VaR revision. In order to check whether the results obtained from the VaR calculation are consistent and reliable; each model must be verified by the so-called *back-testing*, with the help of statistical methods. Brown highlighted the importance of back-testing by saying that „VaR is only as good as its backtest (Brown 2008). When someone shows me a VaR number, I don't ask how it is computed, I ask to see the backtest“. Back-testing is the process where the real gains and losses are compared to the forecasted VaR estimations. If the VaR estimations are not accurate, the model must be reviewed because of incorrect assumptions, the wrong parameters or wrong modelling. Various methods for testing back are proposed. The first test is known as the test of unconditional coverage. What is important to state about this test is that it does not take into consideration when the exception occurred. Another, equally important aspect is to ensure that the observations that exceed VaR are independent, that is, to be equally distributed in time. A “good” model is able to avoid the grouping of deviations, in a way that it quickly responds to changes in the volatility of financial instruments and their correlation (Paseka, Thavaneswaran, Frank 2018). It is known that there are serious problems in the VaR assessments for turbulent markets (Nelson 1991). In fact, by definition, VaR measures the expected losses only under normal market conditions. Good VaR model should give the exact number of deviations (exceptions) that are equally spaced in time, meaning and independent from each other.

2. Methodology and hypothesis

Due to the fact that this research takes into consideration not only one test, the hypothesis for each test will be presented individually. However, the null and alternative hypotheses that summarize five tests for the back-testing of VaR model are simply stated as:

Ho: The VaR model is accurate Ha: The VaR model is not accurate.

The POF test (proportion of failure) is the first test and examines whether the number of exceptions is in accordance with the level of confidence. The null hypothesis for the proportion of failure is expressed as:

$$H0: p = \hat{p} = \frac{X}{T} \quad (2)$$

Variables in formula (2) are p - The proportion of failure, \hat{p} - The observed failure rate, X - Number of exceptions and T - Number of observations. The null hypothesis states that **the observed failure rate \hat{p} is equal to the failure rate which is suggested by the confidence interval**. Furthermore, the goal of accepting the null hypothesis is to prove that the model is accurate. In the case where the amount of likelihood ratio is greater than the critical value of the χ^2 , the conclusion about rejecting the null hypothesis and model inaccuracy would be made. Otherwise, the null hypothesis would be accepted.

The TUFF (time until first failure) is the second test and examines/measures the time until the first exception occurs. The null hypothesis for TUFF test is expressed as:

$$H0: p = \hat{p} = \frac{1}{v} \quad (3)$$

Variables in formula (3) are p - The proportion of failure, \hat{p} - The observed failure rate and v- Time until first exception. The null hypothesis states that **the probability of an exception is the inverse probability of the confidence level for VaR**. The conclusion is the same as for the POF test hypothesis that in the case where the amount of likelihood ratio is greater than the critical value of the χ^2 , the null hypothesis would be rejected and model would be considered as inaccurate. Otherwise, the null hypothesis would be accepted.

The Basel Traffic Light Approach as the third test examines the model accuracy and correctness by measuring the number of exceptions. The null hypothesis states that **the number of exceptions is between 0 and 32 at 90% level of confidence; 0 and 17 at 95%; and between 0 and 4 at 99% level of confidence**. If the number of exceptions does not fall into range, the conclusion about model inaccuracy would be made. Otherwise, the null hypothesis would be accepted. Christoffersen's Independence test,

test number four, examines if the probability of today's exception depends on the outcome from the day before. The null hypothesis is expressed as:

$$H_0: \pi_0 = \pi_1 \quad (4)$$

Where π is probability value. The null hypothesis states that ***an exception today does not depend on whether an exception occurred the day before***. The conclusion is the same as for the POF & TUFF hypotheses that in the case where the amount of likelihood ratio is greater than the critical value of the χ^2 , the null hypothesis about equal distribution of exceptions over time would be rejected and model would be considered as inaccurate. Otherwise, the null hypothesis would be accepted. Mixed Kupiec-test, test number five, proposed by Haas, examines the time between each exceptions, advocating exception independency during the testing period.

The null hypothesis is expressed as:

$$H_0: x_0 = x_1 \quad (5)$$

Where x is the number of exceptions. The null hypothesis for the Mixed Kupiec-test states that ***exceptions are independent of each other over time***. The conclusion is the same as for the the other test hypotheses that in the case where the amount of likelihood ratio is greater than the critical value of the χ^2 , the null hypothesis about exceptions independency over time would be rejected and model would be considered as inaccurate. Otherwise, the null hypothesis would be accepted. The main reason why this research has been done is because there are many discussions whether the VaR models are reliable or not. So, the final purpose of the research is to evaluate through many tests if the respective method for VaR calculation is consistent. This research takes into consideration, for its empirical part, the quantitative research due to the fact that it involves data in numerical form which are further used for statistical calculations in order to draw a conclusion about model accuracy. The empirical part of the research consists of two sections: *calculation of VaR amounts* and *back-testing* those amounts using different types of tests. Data used for those statistical calculations are from secondary source (marketwatch.com) for five Blue-chip companies: Procter & Gamble, Mc Donald's, Microsoft, Caterpillar and Apple as a base for all calculations, simulations and analyses. It is important to emphasize that part of the calculations and graphs are done in excel, and the other part in SPSS, using the adequate formulas or functions. The first step, which is used for both: VaR calculation and back-testing purposes, is to calculate daily returns for each company (without dividends). Daily returns are calculated using the formula for daily returns which is based on daily share closing price. This data calculation is essential for all three VaR calculation methods and further calculations and conclusions.

a) VaR Calculation Methods

Once the daily returns are calculated, the second step for the parametric method (variance-covariance) is to measure the company's standard deviation (volatility) in SPSS. As the name of the method says, variance-covariance, it is necessary to form a covariance matrix as well in SPSS. The covariance matrix is needed in order to calculate the standard deviation of the portfolio. The value of standard deviation, which is calculated from covariance matrix and amount of vector of invested funds, is then used to get the value at risk at 95% level of confidence. All five companies are included in the process.

In order to calculate the VaR according to the historical simulation method, expected daily return for the whole portfolio is mandatory. Expected portfolio daily return is calculated by computing expected daily returns for each company and then summing up the expected daily returns for those five companies. After that, the standard deviation of portfolio's daily returns is measured. The VaR is got by plugging the necessary data in the formula (1) at 90%, 95% and 99% level of confidence. The process includes all five companies.

The third method, Monte Carlo simulation, is only applied for Procter & Gamble and requires the usage of the function *RAND*. By using this function it is possible to get the random numbers. The following formula used in EXCEL is

=NORMINV(RAND();average;standarddeviation), where the amounts of average and standard deviation of the company are previously computed in parametric method. Computation of price change is based on the formula from Geometric Brownian Motion which will be in more detail explained in the Data Analysis part. To calculate VaR at 90%, 95% and 99% level of confidence by using the formula (1), it is required to compute the standard deviation of simulated portfolio's daily returns.

b) Back-testing Process

Since the first test, POF test, requires the number of exceptions which occur at certain level of confidence, it is necessary to compare the estimated VaR with the portfolio's losses. Once the number of exceptions is known, the likelihood ratio test is to be calculated by using the following formula:

$$LR\ POF = -2 \ln \frac{((1-p)^T - x * p^x)}{[1 - (\frac{x}{T})]^{T-x} * (\frac{x}{T})^x} \quad (6)$$

and according to Jorion (2001) the exact definition of the likelihood ratio test is: "Likelihood-ratio test is a statistical test that calculates the ratio between the maximum

probabilities of a result under two alternative hypotheses. The maximum probability of the observed result under null hypothesis is defined in the numerator and the maximum probability of the observed result under the alternative hypothesis is defined in the denominator. The decision is then based on the value of this ratio. The smaller the ratio is, the larger the LR-statistic will be. If the value becomes too large compared to the critical value of χ^2 distribution, the null hypothesis is rejected. According to statistical decision theory, likelihood-ratio test is the most powerful test in its class". In the case where the amount of likelihood ratio is greater than the critical value of the χ^2 , the conclusion about rejecting the null hypothesis and model inaccuracy would be made. The second test, TUFF, requires the time when the first exception occurs, and then by using the following formula, the likelihood ratio at 90%, 95% and 99% level of confidence can be computed:

$$LRTUFF = -2 \ln \frac{(p + (1-p)^{v-1})}{\frac{1}{v} + (1 - \frac{1}{v})^{v-1}} \quad (7)$$

The so-called Christoffersen's Independence Test is the third test which is used for evaluating whether the exceptions that occur are equally spread during the period. This test is for the first step based on contingency table. For the purposes of the contingency table, it is necessary to define two indicators, first a) $It=1 \rightarrow$ if the exception occurs and $It=0 \rightarrow$ if the exception does not occur. This means that if the loss is greater than the forecasted VaR, the assigned indicator would be 1; otherwise 0. From the table below, the "n" stands for the number of days when two conditions (for today: t and for the day before: $t-1$) were met:

Table 1: Contingency Table

	It-1=0	It-1=1	
It=0	n00	n10	n00+n10
It=1	n01	n11	n01+n11
	n00+n01	n10+n11	N

Using the data information from contingency table (table 1), the following formulas are used for computing the probability values:

$$\pi 0 = \frac{n01}{n00+n01} \quad (8)$$

$$\pi 1 = \frac{n11}{n10+n11} \quad (9)$$

$$\pi = \frac{n01+n11}{n00+n01+n10+n11} \quad (10)$$

The formula for likelihood ratio is expressed as:

$$LRind = -2\ln \left(\frac{(1-\pi)^{n00+n10} \cdot \pi^{n01+n11}}{(1-\pi0)^{n00} \cdot \pi0^{n01} \cdot (1-\pi1)^{n10} \cdot \pi1^{n11}} \right) \quad (11)$$

Mixed Kupiec-test for evaluating exception independency is the last test that will be calculated and presented in this research. In order to calculate the required likelihood ratio, tie time between two exceptions is required.

After that, by plugging the necessary data into the same formula as one used for TUFF test:

$$LRTUFF = -2\ln \frac{(p+(1-p)^v)^{v-1}}{\frac{1}{v} \cdot (1-\frac{1}{v})^{v-1}} \quad (12)$$

Once all previously explained calculations are completed, the comparison of results obtained and corresponding critical values is done. The comparison is done for each test. These comparisons are necessary in order to make correct conclusions about the validity of the model.

4. DATA ANALYSIS AND DISCUSSION

The empirical part of this research is divided into two sections: VaR calculation methods and back-testing process.

a) VaR Calculation Methods

As is already written in the literature review and methodology part, there are three methods for calculating forecasted Value at Risk.

➤ The first one is the Parametric method or the so-called Variance Covariance model. This model considers that all changes in the market are normally distributed and in order to apply this method, the set of simple data information is required. So, because of this, for calculation purposes, set of daily close prices are observed for five companies. The first step for the calculation of VaR using the parametric method is to collect the data for five companies and their closing prices during the period.

Table 2: Daily prices

Date	Price				
	Procter&Gamble	McD	Microsoft	Caterpillar	Apple
8/4/2014	79.22	94.31	43.37	101.81	95.59
8/1/2014	79.65	94.30	42.86	100.52	96.13
7/31/2014	77.32	94.56	43.16	100.75	95.60
7/30/2014	78.16	95.95	43.58	103.38	98.15
7/29/2014	78.65	95.82	43.89	104.69	98.38
		.			
		.			
8/15/2013	80.48	95.39	31.79	85.86	71.13
8/14/2013	81.25	96.11	32.35	85.82	71.21
8/13/2013	81.66	96.45	32.23	86.57	69.94
8/12/2013	81.62	97.04	32.87	86.32	66.77
8/9/2013	81.64	97.62	32.70	84.51	64.92
8/8/2013	82.17	98.04	32.89	83.96	65.86
8/7/2013	81.96	98.33	32.06	82.43	66.43
8/6/2013	81.74	98.69	31.58	82.53	66.46
8/5/2013	81.40	99.31	31.70	83.56	67.06

Once the daily share prices are collected from the official Yahoo Finance page, daily returns for each company are calculated in EXCEL using the formula for daily return:

$$r = \ln \frac{St}{St-1} \quad (12)$$

Variables in formula (12) are:

r - Daily return

ln – Natural logarithm

St- Price today

St-1 - Price day before

Daily returns of each company are presented in the table below:

Table 3: Daily Returns

<i>Returns (%)</i>				
Procter&Gamble	McD	Miscrosoft	Caterpillar	Apple
-0.541%	0.011%	1.183%	1.275%	-0.563%
2.969%	-0.275%	-0.698%	-0.229%	0.553%
-1.081%	-1.459%	-0.965%	-2.577%	-2.632%
-0.625%	0.136%	-0.701%	-1.259%	-0.234%
-0.773%	0.042%	-0.194%	0.517%	-0.648%
-0.378%	0.063%	-1.198%	-0.670%	1.372%
0.874%	0.258%	0.407%	0.459%	0.633%
⋮				
-0.189%	-0.409%	-0.032%	-1.103%	0.257%
-0.075%	0.021%	0.720%	-0.405%	-1.322%
-0.389%	0.472%	-1.288%	-1.134%	1.071%
-0.647%	-0.429%	-0.579%	0.653%	-1.433%
0.256%	-0.295%	2.547%	1.839%	-0.857%
0.269%	-0.365%	1.518%	-0.121%	-0.058%
0.417%	-0.626%	-0.379%	-1.240%	-0.899%

As the daily returns (table 3) for each company are calculated and the results are presented through the graphs, the values of standard deviation (volatility) for daily returns are calculated in SPSS. It can be seen, since the standard deviation of each company is close to 0, that daily returns tend to be very close to the mean. The results are presented in the table 4.

Table 4: Standard deviation

Descriptive Statistics					
	N	Minimum	Maximum	Mean	Std. Deviation
Procter& Gamble	251	-.0212049830	.0296893820	-.000108153310	.0078917720328
McD	251	-.0146537679	.0369152133	-.000205812919	.0068952041537
Microsoft	251	-.0465770461	.0703298388	.001248809870	.0136383321064
Caterpillar	251	-.0625890936	.0577191633	.000787025476	.0112286341552
Apple	251	-.0833024982	.0787942579	.001412017339	.0145728769847
Valid N (listwise)	251				

The next step is forming the variance/covariance matrix based on daily returns, which are presented in Table 5. The variance/covariance matrix output is formed in SPSS indicating that both company's daily returns are increasing or decreasing together and can be seen in Table 5:

Table 5: Covariance Matrix

Inter-Item Covariance Matrix

	P&G Return	McD Return	Microsoft Return	CAT Return	Apple Return
Procter& GambleReturn	.0000623	.0000189	.0000297	.0000116	.0000075
McD Return	.0000189	.0000475	.0000176	.0000270	.0000028
Microsoft Return	.0000297	.0000176	.0001860	.0000437	.0000067
Caterpillar Return	.0000116	.0000270	.0000437	.0001261	.0000243
Apple Return	.0000075	.0000028	.0000067	.0000243	.0002124

These values from the matrix are necessary for the calculation of standard deviation of the portfolio in a way that the covariance matrix is multiplied by the value of vector of invested funds which in this case is taken as \$2000:

$$\begin{pmatrix} 0.00006 & 0.00002 & 0.00003 & 0.00001 & 0.00001 \\ 0.00002 & 0.00005 & 0.00002 & 0.00003 & 0.00000 \\ 0.00003 & 0.00002 & 0.00019 & 0.00004 & 0.00001 \\ 0.00001 & 0.00003 & 0.00004 & 0.00013 & 0.00002 \\ 0.00001 & 0.00000 & 0.00001 & 0.00002 & 0.00021 \end{pmatrix} * \$2000 * \$2000 * \$2000 * \$2000 * \$2000$$

$$= \begin{pmatrix} 0.00013 \\ 0.00011 \\ 0.00028 \\ 0.00023 \\ 0.00025 \end{pmatrix} * \$2000 * \$2000 * \$2000 * \$2000 * \$2000$$

After the matrix vector is being calculated, it is multiplied by the transposed vector of invested funds:

$$= \begin{pmatrix} 0.00013 \\ 0.00011 \\ 0.00028 \\ 0.00023 \\ 0.00025 \end{pmatrix} \begin{matrix} \$2000 \\ \$2000 \\ * \$2000 \\ \$2000 \\ \$2000 \end{matrix} = \$2.03$$

Using the *STDEV* formula in EXCEL, the standard deviation (volatility) of portfolio is calculated: $\sigma = \sqrt{2.03} = \$1.42$

Now, having the amount of standard deviation of portfolio, the Value at Risk can be calculated at the 90%, 95% and 99% level of confidence. Value at Risk at these levels of confidence equals to the product of standard deviation and confidence level, which in

this case gives the following result suggesting that not more than this amount would be lost at the respective level of confidence:

$$\text{VaR } 90\% = \$1.42 * (-1.65) = \mathbf{-\$2.35} \quad \text{VaR } 95\% = \$1.42 * (-1.96) = \mathbf{-\$2.79} \quad \text{VaR } 99\% = \$1.42 * (-2.58) = \mathbf{-\$3.67}$$

➤ The second method for calculating VaR is called the method of historical simulation. Historical simulation is a non-parametric method.

For the purposes of calculating VaR, the same data are necessary as for the parametric method: prices for five companies are considered for the period. Daily returns for each one of five companies are calculated in a same way as for the parametric method, but now once daily returns for each company are calculated, the expected daily return for the whole portfolio is calculated by computing expected daily returns for each company and then summing up the expected daily returns for those five companies. Expected daily returns for the company are calculated in a following way: daily return (%) * amount of invested funds in portfolio. As an example, it is taken that the amount of invested funds in portfolio equals to \$2000. By using the method for obtaining the expected daily returns for each company and for the whole portfolio, Table 6 shows the results:

Table 6: Expected Daily Returns

Expected Daily Returns (\$)					
Procter&Gamble	McD	Microsoft	Caterpillar	Apple	Portfolio
-10.83	0.21	23.66	25.50	-11.27	27.28
59.38	-5.51	-13.95	-4.57	11.06	46.41
-21.61	-29.19	-19.30	-51.54	-52.65	-174.28
-12.50	2.71	-14.02	-25.18	-4.68	-53.67
-15.45	0.84	-3.87	10.34	-12.97	-21.11
-7.56	1.25	-23.96	-13.40	27.43	-16.23
-6.72	-29.31	6.48	1.09	-10.43	-38.88
10.15	-0.99	25.25	4.79	23.62	62.82
⋮					
9.80	7.35	48.75	29.44	2.39	97.73
-3.78	-8.18	-0.63	-22.06	5.14	-29.51
-1.51	0.42	14.41	-8.09	-26.45	-21.22
-12.94	-8.59	-11.59	13.06	-28.67	-48.72
5.12	-5.91	50.93	36.78	-17.15	69.78
5.38	-7.31	30.36	-2.42	-1.16	24.84
8.34	-12.53	-7.59	-24.81	-17.97	-54.55

Once the expected daily returns are calculated, the next step is forming the correlation matrix for the share prices from which the degree, to which shares from companies move in relation to each other, is visible. The correlation matrix is calculated in order to

get the coefficient of correlation based on the share prices movements. Table 7 presents the calculated coefficients whose values fall between -1 and 1:

Table 7: Correlation Matrix

		Correlations				
		P&G	McD	Microsoft	Caterpillar	Apple
P&G	Pearson Correlation	1	.345	.218	-.031	.167
McD	Pearson Correlation	.345	1	.509	.663	.524
Microsoft	Pearson Correlation	.218	.509	1	.857	.858
Caterpillar	Pearson Correlation	-.031	.663	.857	1	.799
Apple	Pearson Correlation	.167	.524	.858	.799	1

Since now the expected daily returns of portfolio are known, the results can be used for further calculations. First of all the standard deviation (volatility) of the portfolio and the amount of average expected daily returns of the portfolio are calculated in SPSS:

Table 8: Average Return and Standard Deviation

Descriptive Statistics			
	N	Mean	Std. Deviation
Portfolio	251	6.267772905	63.68454042
Valid N (listwise)	251		

These amounts are necessary for one day VaR estimation which is done in EXCEL using the formula (1) and taking the value of initial portfolio value (amount of invested fund) to be equal to \$2000. So, having all the necessary data, by plugging it in the formula (1), the VaR values are shown in Table 9:

Table 9: VaR Amounts

Confidence Level	1-day VaR
90%	-6.63
95%	-7.88
99%	-10.37

➤ The third method for VaR calculation is Monte Carlo Simulation, which is also not a parametric method as a historical simulation. However, it is a way more complex to implement. This method will be applied for one Proctor & Gamble company. Since daily

Each time the simulation is started, new values and results are available. So, one of the possible simulated scenarios represents the share prices movement for the future period of 251 days. After the simulation is finished, the standard deviation of the simulated daily returns is needed to be calculated in order to get the VaR for three levels of confidence. Again, by using the SPSS, standard deviation equals to:

Table 11: Simulated Portfolio Standard Deviation

Descriptive Statistics			
	N	Mean	Std. Deviation
Returns	251	.000933709	.0079907121
Valid N (listwise)	251		

By using the formula (1) and new standard deviation of the simulated daily returns for the company Procter& Gamble, VaR equals to amounts as shown in Table 12:

Table 12: VaR Amounts

Confidence Level	1-day VaR
90%	-0.094
95%	-0.111
99%	-0.147

The process of simulation can be performed as many as possible times, and each time the VaR calculations will be automatically updated.

b) Back-testing Process

The process of back-testing is the essential part when making any conclusion whether the model is „good“ or „bad“, reliable or not. For the purposes of this research, the back-testing process is done for the calculated VaR forecast using the historical simulation method. There are many methods for the so-called *back-testing*. The most basic and most commonly used tests are divided into two groups:

1) *Unconditional coverage* is the first group that consists of three tests:

➤ **Kupiec's POF** test stands for the proportion of failure, and measures whether the number of exceptions is in accordance with the level of confidence. The first step in

testing is to calculate daily returns for each company, daily returns for the whole portfolio of five companies, average return of the portfolio and standard deviation of the portfolio. These calculations are already done for the purposes of estimating VaR amounts, so the same are going to be used for the back-testing process. Since the POF test as an essential part considers the number of exceptions, it is necessary to calculate how many exceptions occur. In order to get the number of exceptions which occurs for each level of confidence, the daily losses of portfolio are observed and then compared to the calculated (forecasted) VaR. The exception is present if the value of loss is greater than the forecasted VaR value. Table 13 shows the part of results for examining whether the exception or even the profit occurs:

Table 13: Exception occurrence

1 day VaR 99%	Exception	1 day VaR 95%	Exception	1 day VaR 90%	Exception
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	Profit	-7.88	Profit	-6.63	Profit
-10.37	NO	-7.88	NO	-6.63	NO
-10.37	YES	-7.88	NO	-6.63	NO

The described process in the paragraph above and its results are summarized in Table 14:

Table 14: One day VaR & Exceptions

Confidence Level	1-day VaR	Exceptions
90%	-6.63	9
95%	-7.88	11
99%	-10.37	12

After the number of exceptions is known for each level of confidence, the process of back-testing can start which in this case is the process of evaluating the POF test. The information necessary for the POF test calculation are presented in the table 15:

Table 15: POF Test Data

Level of confidence	Number of observations	Number of exceptions
90%	251	9
95%	251	11
99%	251	12

As is already stated earlier, for the POF test the calculation of likelihood test is needed. The likelihood test is expressed through the formula (6). So, now the corresponding likelihood ratio test can be calculated by plugging the appropriate data from Table 5 in the formula for likelihood ratio. For each one of three LR calculations for back-testing purposes, 95% is taken as the critical value. In other words this means that the strong evidence is required for rejecting the null hypothesis and model accuracy. For the purposes of making a valid conclusion about the model accuracy, the critical value from the well-known table called *Chi-Squared Distribution* is used (Table 16).

Test 1:

The portfolio with 9 exceptions (9 times the amount of portfolio's daily returns/losses is greater than the estimated VaR calculation) is taken into consideration at the 90% confidence level during the 251 trading/working days. First of all, the critical value is taken from the Critical Value χ^2 for the Chi-Squared Distribution (for **95%** confidence interval, critical value is **3.84**):

Table 16: Chi-Squared Distribution

f	p value												
	0.995	0.99	0.975	0.95	0.9	0.75	0.5	0.25	0.1	0.05	0.025	0.01	0.005
1	0.00	0.00	0.00	0.00	0.02	0.10	0.45	1.32	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	0.21	0.58	1.39	2.77	4.61	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	0.58	1.21	2.37	4.11	6.25	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	1.06	1.92	3.36	5.39	7.78	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	1.61	2.67	4.35	6.63	9.24	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	2.20	3.45	5.35	7.84	10.64	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.22	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.39	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.55	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	7.58	10.34	13.70	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.85	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	9.30	12.34	15.98	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	10.17	13.34	17.12	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	11.04	14.34	18.25	22.31	25.00	27.49	30.58	32.80

The likelihood ratio test in this case equals to:

$$\text{LR POF} = -2 \ln \frac{((1-0.10)^{251-9} * 0.10^9)}{\left[1 - \left(\frac{9}{251}\right)\right]^{251-9} * \left(\frac{9}{251}\right)^9} = 14.85$$

Test 2:

Furthermore, the portfolio with 11 exceptions (11 times the amount of portfolio's daily returns/losses is greater than the estimated VaR calculation) is taken into consideration at the 95% confidence interval during the 251 trading/working days. First of all, the critical value is taken from the Critical Value χ^2 for the Chi-Squared Distribution (for 95% confidence interval \rightarrow critical value is 3.84) same as in the previous example. The likelihood ratio test in this case equals to:

$$\text{LR POF} = 2 \ln \frac{((1-0.05)^{251-11} * 0.05^{11})}{\left[1 - \left(\frac{11}{251}\right)\right]^{251-11} * \left(\frac{11}{251}\right)^{11}} = 0.21$$

Finally, for the third case the portfolio with 12 exceptions (12 times the amount of portfolio's daily returns/losses is greater than the estimated VaR calculation) is taken into consideration at the 99% confidence interval during the 251 trading/working days. First of all, the critical value is taken from the Critical Value χ^2 for the Chi-Squared Distribution (for 95% confidence interval \rightarrow critical value is 3.84) same as in the previous two examples. The likelihood ratio test in this case equals to:

$$\text{LR POF} = -2 \ln \frac{((1-0.01)^{251-12} * 0.01^{12})}{\left[1 - \left(\frac{12}{251}\right)\right]^{251-12} * \left(\frac{12}{251}\right)^{12}} = 18.94$$

➤ **Kupiec's TUFF** measures “the time it takes for the first exception to occur” (Kupiec 1995). So, because of this it is necessary to measure the number of days until the first exception. These values for each level of confidence (90%, 95% and 99%) are measured from the Table 14: Exception occurrence. The results are summarized in Table 17:

Table 17: Number of days until first exception

Confidence Level	Number of days until first exception
90%	43
95%	43
99%	29

The second step in the PUFF test is to calculate the Likelihood ratio, by plugging in the necessary data into the formula (7):

Test 1:

For the test 1, the likelihood ratio is calculated at the 90% level of confidence. In this case, the amount of „p“ equals to 0.1 (p=1-0.90). The likelihood ratio equals to:

$$\text{LRTUFF} = 2 \ln \frac{(0.1 * (1 - 0.1)^{43-1})}{\frac{1}{43} * \left(1 - \frac{1}{43}\right)^{43-1}} = 3.9564$$

Test 2:

For the test 2, the likelihood ratio is calculated at the 95% level of confidence. In this case, the amount of „p“ equals to 0.05 (p=1-0.95). The likelihood ratio equals to:

$$\text{LRTUFF} = 2 \ln \frac{(0.05 * (1 - 0.05)^{43-1})}{\frac{1}{43} * \left(1 - \frac{1}{43}\right)^{43-1}} = 0.8011$$

Test 3:

For the test 3, the likelihood ratio is calculated at the 99% level of confidence. In this case, the amount of „p“ equals to 0.01 ($p=1-0.99$). The likelihood ratio equals to:

$$\text{LRTUFF} = 2 \ln \frac{(0.01 * (1 - 0.01)^{251-1})}{\frac{1}{251} * (1 - \frac{1}{251})^{251-1}} = 1.0734$$

➤ **Basel Traffic Light Approach** uses the same data as the Kupiec's POF test uses for making a conclusion about VaR model accuracy: number of exceptions at each level of confidence for the period of 251 working/trading days. This information is already presented in the POF test analysis, table 18:

Table 18: Basel Traffic Light Approach

Level of confidence	Number of observations	Number of exceptions
90%	251	9
95%	251	11
99%	251	12

2.) *Conditional coverage* is the second group that consists of two tests:

➤ **Christoffersen's Independence Test** is applied in order to examine if the exceptions are equally distributed in time period. For the purposes of this test, the contingency table, which is already explained in the Methodology part, has to be set up. By using the available and necessary data, in this case at the 90% level of confidence, the contingency table is presented below (tables 19 to 21)

Table 19: Contingency Table at 90%

	It = 0	It = 1	
It = 0	233	9	242
It = 1	9	0	9
	242	9	251

Table 20: Contingency Table at 95%

	It = 0	It = 1	
It = 0	230	10	240
It = 1	10	1	11
	240	11	251

Table 21: Contingency Table at 99%

	It = 0	It = 1	
It = 0	228	11	239
It = 1	11	1	12
	239	12	251

Now, once the contingency tables are set up for 90%, 95% and 99%, the probability values π_0 , π_1 , π_2 can be calculated by plugging in the data from contingency tables into the formulas (8, 9, 10):

90% level of confidence:

$$\pi_0 = \frac{n_{01}}{n_{00}+n_{01}} = \frac{9}{9+233} = 0.037190$$

$$\pi_1 = \frac{n_{11}}{n_{10}+n_{11}} = \frac{0}{9} = 0$$

$$\pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}} = \frac{9+0}{233+9+9+0} = 0.035856574$$

95% level of confidence:

$$\pi_0 = \frac{n_{01}}{n_{00}+n_{01}} = \frac{10}{240+10} = 0.041666667$$

$$\pi_1 = \frac{n_{11}}{n_{10}+n_{11}} = \frac{1}{11} = 0.090909091$$

$$\pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}} = \frac{10+1}{230+10+10+1} = 0.043824701$$

99% level of confidence:

$$\pi_0 = \frac{n_{01}}{n_{00}+n_{01}} = \frac{11}{228+11} = 0.046025105$$

$$\pi_1 = \frac{n_{11}}{n_{10}+n_{11}} = \frac{1}{12} = 0.083333$$

$$\pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}} = \frac{11+1}{228+11+11+1} = 0.047809$$

Table 22: Independence Test

<i>Back-testing Data Results for the Independence Test</i>								
Confidence Level	Number of exceptions	n00	n01	n10	n11	π_0	π_1	π
90%	9	233	9	9	0	0.037190	0	0.035857
95%	11	230	10	10	1	0.041667	0.090909	0.043825
99%	12	228	11	11	1	0.046025	0.083333	0.047809

The calculations are summarized in the table 22. The final step is to calculate the Likelihood ratio by applying the formula (11).

Test 1:

$$LR_{ind} = -2\ln \left(\frac{(1-\pi)^{n_{00}+n_{10}} \cdot \pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}} \cdot \pi_0^{n_{01}} \cdot (1-\pi_1)^{n_{10}} \cdot \pi_1^{n_{11}}} \right) =$$

$$LR_{ind} = -2\ln \left(\frac{(1-0.035857)^{233+9} \cdot 0.035857^{9+0}}{(1-0.037190)^{233} \cdot 0.037190^9 \cdot (1-0)^9 \cdot 0^0} \right) = 0.6695$$

Test 2:

$$LR_{ind} = -2\ln \left(\frac{(1-0.043825)^{230+10} \cdot 0.043825^{10+1}}{(1-0.041667)^{230} \cdot 0.041667^{10} \cdot (1-0.090909)^{10} \cdot 0.090909^1} \right) = 0.4765$$

Test 3:

$$LR_{ind} = -2\ln \left(\frac{(1-0.047809)^{228+11} \cdot 0.047809^{11+1}}{(1-0.046025)^{228} \cdot 0.046025^{11} \cdot (1-0.083333)^{11} \cdot 0.083333^1} \right) = 0.2916$$

➤ **Mixed Kupiec Test** was suggested by Haas (Haas 2001) and as a first step it takes into consideration the time between two exceptions.

Test 1:

At 90% level of confidence (which is already calculated and presented in Table 13: One day VaR & Exceptions), there are 9 exceptions that occur, meaning that 9 times the value of loss exceeded the estimated VaR amount. Table 23 shows the time between each exception at 90% level of confidence:

Table 23: Time between exceptions (at 90%)

	Number of the exception								
	1	2	3	4	5	6	7	8	9
Time between two exceptions	43	6	12	9	19	25	4	12	52

Now, by applying the formula, it is possible to get the values of likelihood ratio at 90% level of confidence for each exception as shown in Table 24.

Table 24: LR statistics for exceptions (at 90%)

	Number of the exception								
	1	2	3	4	5	6	7	8	9
LR statistics	3.96	0.25	0.04	0.01	0.56	1.27	0.74	0.04	5.47

The final step is to sum up the LR statistics for all exceptions by using the formula:

$$\text{LRind} = 8.26+0.25+0.04+0.01+0.56+1.27+0.74+0.04+5.47 \rightarrow = 12.33$$

Test 2:

At 95% level of confidence, 11 exceptions occur as shown in Table 25.

Table 25: Time between exceptions (at 95%)

	Number of the exception										
	1	2	3	4	5	6	7	8	9	10	11
Time between two exceptions	43	6	12	9	0	14	3	25	4	12	52

After plugging in the data from the previous table into the Formula 12, the computed likelihood ratios at 95% confidence level for each time the exception occurred is in Table 26:

Table 26: LR statistics for exceptions (at 95%)

	Number of the exception										
	1	2	3	4	5	6	7	8	9	10	11
LR statistics	0.80	1.10	0.24	0.53	0.00	0.12	2.38	0.06	1.80	0.24	1.34

The final step is to sum up the LR statistics for all exceptions by using the formula

$$\text{LRind} = 0.80+1.10+0.24+0.53+0+0.12+2.38+0.06+1.80+0.24+1.34 \rightarrow = 8.60$$

Test 3: At 99% level of confidence, 11 exceptions occur. Time between each exception is presented below in Table 27.

Table 27: Time between exceptions (at 99%)

	Number of the exception											
	1	2	3	4	5	6	7	8	9	10	11	12
Time between two exceptions	29	13	6	12	9	0	14	3	25	4	12	52

As time between two exceptions is known, the next step is to calculate the likelihood ratio at 99% level of confidence:

Table 28: LR statistics for exceptions (at 99%)

	Number of the exception											
	1	2	3	4	5	6	7	8	9	10	11	12
LR statistics	1.07	2.40	3.90	2.55	3.09	0	2.27	5.43	1.30	4.77	2.55	0.35

Finally we sum up the LR statistics from table 28 for all exceptions by using the formula

$$\text{LRind} = 1.07+2.40+3.90+2.55+3.09+0+2.27+5.43+1.30+4.77+2.55+0.35 \rightarrow = 26.68$$

5. RESULTS and CONCLUSION

POF Test Results Discussion

The calculated likelihood ratio at 90% and 99% confidence level is in a great amount larger than the critical value. More precisely, since the calculated value 14.86 of likelihood test for the portfolio is greater than the critical value ($p=1-c \rightarrow p=1-0.95=0.05$) 3.84; the statistic test shows that the model is rejected at the 90% level of confidence. This means that the test outcome shows that the null hypothesis, which says that the model is "good", is rejected with 90% of confidence. The same is with the example for 99% level of confidence: the calculated amount of LR is 18.94 and is greater than the critical value. In other words, by calculating the likelihood ratio for levels 90% and 99% of confidence, it is identified that the observed rates of failure are different from the suggested by the confidence interval rate of failure. For these two tests (Test 1 and Test 3) it can be said as well that the VaR estimation underestimates the risk. This is not the case with the portfolio at 95%, where the calculated LR value is equal to 0.21 which is lower than the critical value. This indicates that the test outcome is to accept the model at 95% of confidence. The best overview of results is drawn in Table 29 which summarizes the calculated values for POF test at three confidence levels:

Table 29: POF test results

Kupiec's POF Test			
Confidence for Portfolio	Test statistics LR POF	Critical value $\chi^2(1)$	Test Outcome
90%	14.86	3.84	Reject
95%	0.21	3.84	Accept
99%	18.94	3.84	Reject

TUFF Test Results Discussion

The calculated likelihood ratio at 95% and 99% level of confidence is a way below the critical value ($p=1-c \rightarrow p=1-0.95=0.05$) 3.84. In other words, calculated values of 0.80 and 1.07 are lower than the critical value. This indicates that at the 90% and 99% level of confidence, the null hypothesis stating that the probability of an exception is the inverse probability of the confidence level for VaR is accepted. Since the null hypothesis is not rejected, that means the model is considered as accurate. On the other hand, at 90% level of confidence the calculated likelihood ratio equals to 3.96 and is great than critical value for 0.12. Even though the difference is not significantly above the critical value, the model is rejected with 90% of confidence. The summary of TUFF test results are shown in Table 30:

Table 30: TUFF test results

Kupiec's TUFF Test			
Confidence Level	Test statistics LR TUFF	Critical value $\chi^2(1)$	Test Outcome
90%	3.96	3.84	Reject
95%	0.80	3.84	Accept
99%	1.07	3.84	Accept

Basel Traffic Light Results Discussion

The approach suggests the underestimation of risk at 99% level of confidence, since the number of observations equals to 12 which is according to the Basel Committee inaccurate model. Test does not have results in a yellow zone which would indicate a possible problem with the model. However, at 90% level of confidence number of exceptions equals to 9 which falls between 0 and 32 resulting with the green zone. The same is with the 11 exceptions at 95% level of confidence: model is in the green zone. Even though results are indicating a green zone model, more tests are needed in order to discuss the quality and accordance of the model. Summarized results and zones are shown in Table 31:

Table 31: Basel Traffic Light test results

Basel Traffic Light			
Level of confidence	Number of observations	Number of exceptions	Test results
90%	251	9	Green zone
95%	251	11	Green zone
99%	251	12	Red zone

Christoffersen's Independence Test Results Discussion

The calculated likelihood ratios at 90%, 95% and 99% confidence level are a way below the critical value indicating that the null hypothesis, which states that an exception today does not depend on whether an exception occurred the day before, cannot be rejected. Based on the results, it can be said that no dependence is present between the exceptions at all of three confidence levels. Test outcomes are summarized in Table 32:

Table 32: Christoffersen's Independence test results

Christoffersen's Test			
Confidence Level	Test statistics LRind	Critical value $\chi^2(1)$	Test Outcome
90%	0.67	3.84	Accept
95%	0.48	3.84	Accept
99%	0.29	3.84	Accept

Mixed Kupiec Test - Independence Test Results Discussion

The critical values used for Mixed Kupiec-test are assigned by the number of exceptions for each level of confidence. As can be seen from the table below, the calculated likelihood ratio at 99% level of confidence exceeds the critical value by 8.65. Since the calculated value is greater than the critical value, the null hypothesis is rejected. By rejecting the null hypothesis, the conclusion about model inaccuracy, indicating that the independence property is not satisfied. Moreover, at 90% and 95% level, test statistics for likelihood ratios are smaller than the assigned critical values and the model is accepted indicating that exceptions are independent of each other as summarized in Table 33.

Table 33: Mixed Kupiec-Test: Independence test results

Independence Test - Mixed Kupiec-Test			
Confidence Level	Test statistics LR	Critical value χ^2	Test Outcome
90%	12.33	16.92	Accept
95%	8.60	19.68	Accept
99%	29.68	21.03	Reject

Finally summarized results from all tests can be seen in Table 34:

Table 34: Summarized results

Confidence Level	Observations	Exceptions	POF Test	TUFF Test	Traffic Light Approach	Christoffersen's Test	Mixed Kupiec-Test
90%	251	9	Reject	Reject	Green zone	Accept	Accept
95%	251	11	Accept	Accept	Green zone	Accept	Accept
99%	251	12	Reject	Accept	Red zone	Accept	Reject

It is well-known that the usage of VaR forecast is widespread. Since there is no such a method which predicts the accurate forecast, certain back-testing procedures should be undertaken in order to evaluate whether the calculated VaR results are satisfactory or not. Back-testing is definitely a necessity; however, more backtests should be done to confirm the accuracy and reliability of the VaR model validation. This fact indicates that the back-testing should be a part of daily VaR calculations. The results from back-testing are able to provide information whether potential problems or risks exist in the company's core system, so in that way the company's management can take necessary risk mitigation measures and protect company against the potential future risk.

In this research using secondary data daily share prices, daily returns for each company and daily returns of the entire portfolio of five companies during the period were taken as crucial parameters.

Back-testing process should be the essential part of reporting regulation in every financial institution in order to be sure that the current VaR measure technique is ensuring consistent forecasts.

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