A time-heterogeneous generalised Pareto distribution for tail risk estimation in the energy market

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Abstract. *Price volatility poses a threat to financial stability and investment strategies in the South African energy sector. This calls for accurately evaluating extreme financial risk, which helps in understanding and managing the risks associated with rare events.* ***Objective:*** *This study aims to assess the severity and probability of extreme losses in the South African energy sector by dynamically modelling tail behaviour using time-varying parameters.* ***Methods:*** *Using a time-heterogeneous generalised Pareto distribution (GPD), the study scrutinises downside risk in the FTSE/JSE Oil, Coal and Gas Index (FTSE/JSE-J500) for the period January 8, 2012, to February 4, 2025. Threshold selection is performed using the Inverse Hill Statistic and the ForwardStop algorithm for time-varying and fixed thresholds, respectively, and model parameters are estimated via the weighted nonlinear least squares likelihood-moments method.* ***Results:*** *The empirical findings indicate a heavy-tailed returns distribution, as evidenced by a positive shape parameter. The value-at-risk (VaR) is expected to be 0.3296, while the expected shortfall (ES) is approximately 0.4058, at the 99.9% confidence level. The backtesting results validate the efficacy of the time-heterogeneous tail risk model in improving predictive accuracy and resilience in volatile markets. The study further indicates that utilising a dynamic threshold method enhances the characterisation of extreme losses and provides a more accurate basis for financial risk management.* ***Originality:*** *The originality of this study lies in its ability to dynamically model the time-varying behaviour of extreme losses in the South African energy sector using a time-heterogeneous Generalised Pareto Distribution. By incorporating time-varying thresholds via the Inverse Hill Statistic and applying weighted nonlinear least squares likelihood-moment estimation, the study captures evolving downside risk with greater precision than static extreme-value approaches. This work is among the few to analyse extreme price movements in the FTSE/JSE Oil, Coal and Gas Index over an extended period, including multiple episodes of volatility.*

Keywords*: Backtesting, Dynamic Modelling, Energy Market, Extreme Value Theory, Financial Stability, Threshold Selection, South Africa, weighted nonlinear least squares likelihood moments*
JEL Classifications: *C13, C52, C58*

1. Introduction

Coal remains the cornerstone of South Africa's energy industry, accounting for most of the nation's electrical output. Efforts to diversify the energy mix through greater investment in renewables such as solar and wind are underway, but coal is likely to remain dominant in the near future. Persistent load shedding (regular, planned power outages) highlights the urgent need for major investment in

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energy infrastructure, a more balanced energy portfolio, and stronger financial risk management strategies. Currently, risk management in the South African energy sector centres on operational strategies like reserve margin planning, contingency measures for load shedding, and financial hedging through energy futures and derivatives. While these approaches address common operational and market risks, the OECD (2025) notes that they largely depend on historical data and are not designed to anticipate extraordinary market events or unexpected volatility. This study, therefore, introduces a statistical application of extreme value theory, employing a time-varying Generalised Pareto Distribution (GPD) for South African energy stock prices. This enables market participants to better assess tail risk, enhance stress testing, and implement proactive hedging strategies to strengthen financial and operational resilience during volatility.

Nevertheless, financial markets have become increasingly complex and expansive in the contemporary global economy. The COVID-19 pandemic disrupted global economic activity and raised concerns about a recession similar to 2008–2009, as Shaik and Padmakumari (2022) indicate. During this period, the South African financial sector faced disruptions from both global and domestic shocks (Naradh et al, 2021), closely linked to evolving phases of the pandemic in the country, where the emergence of the Delta and Omicron variants intensified market volatility and investor uncertainty. In response, aligning with international public health strategies, the South African government declared a national state of disaster. While this step was crucial for managing the health crisis, it brought significant economic consequences, including suppressed growth, elevated volatility, investment losses, and reduced consumer confidence. Hence, understanding and responding quickly to these external shocks is crucial in managing financial market risks. To inform risk assessment and decision-making, accurate and robust models are essential. The GARCH (herein referenced generalised autoregressive conditional heteroscedasticity) model is a standard tool for modelling volatility (Jafari et al, 2007); however, it has been criticised for its limitations in addressing structural shifts and asymmetries, as emphasised by Mzamane et al. (2013). As such, models with time-varying parameters better reflect the evolving behaviour of market volatility, especially when influenced by macroeconomic factors and investor sentiment, and such dynamics, according to Makatjane et al. (2025), amplify market swings and losses, particularly during economic stress.

To overcome this problem, we introduce in this study an integrated non-stationary peaks-over-threshold (POT) approach within extreme value theory. This approach links a dynamic, automated time-varying threshold selection method, using the inverse Hill statistic, to a two-parameter Generalised Pareto Distribution. By overcoming outmoded threshold-selection limitations that assume constant tail behaviour, our dynamic approach improves stability in tail-risk estimation. This adaptation aligns with the Adaptive Market Hypothesis (AHM), which recognises evolving efficiency in response to economic and behavioural shifts, rather than static market behaviour. Moreover, to ensure that our modelling approach produced robust and reliable parameter estimates, we then use a weighted nonlinear least squares (WNLS) approach, which is central to this study. Unlike standard nonlinear least squares, the WNLS enables flexible adaptation to time variation and tail risk by emphasising extreme observations. In comparison with other estimation methods, maximum likelihood estimation (MLE) encounters frequent problems with time-varying GPDs, including unbounded likelihoods over certain parameter ranges, sensitivity to initial values, convergence failures, and pronounced bias and variance in small samples (Liyin et al., 2025). These problems intensify with more complex model structures and greater computational demands. Additionally, the MLE's vulnerability to outliers and its reliance on subjective thresholds worsen bias–variance trade-offs. Because GPD models often assume stationarity, they struggle to account for sudden changes in market dynamics and time-varying tail behaviour.

When it comes to the Bayesian approaches, their advantages and limitations in dynamic extreme value modelling deserve careful consideration. These approaches with respect to Lee (2024) offer comprehensive probabilistic inference, but often suffer from computational inflexibility in time-varying contexts, requiring a complex hierarchy and repeated posterior sampling. Furthermore, challenges with defining appropriate priors as indicated by Haddad (2025) persist, especially in unstable environments, which biases results or complicates interpretation (de-Carvalho et al., 2025). By contrast, WNLS delivers practical adaptability and clarity, making it especially fit for modelling dynamic tail behaviour in financial and extreme-value applications.

Lastly, the suggested method for selecting a time-varying threshold is based on a test of the exponential approximation for log-spaced data. We therefore calculate the integrated squared error of the exponential density, assuming that the log spacing is exponentially distributed, and use the inverse Hill statistic (IHS) as the error measure. Because the IHS is easy to calculate, we use it because it requires no tuning parameters, and can be smoothed to improve performance in small-sample settings. Its minimising oracle sample fraction is asymptotically smaller than that of the original Hill estimator, indicating heightened sensitivity to deviations from exponentiality; yet it performs exceptionally well in finite samples. To establish the sufficiency of the proposed context, we conduct comprehensive statistical analyses, including cross-comparative studies of long and short trading positions across positive and negative returns. These analyses evaluate the hypothesis that adaptive, time-varying threshold selection utilising the inverse Hill statistic yields more stable and robust tail estimates than conventional methods, such as Hill plots, mean excess functions, and current historical or adaptive threshold selection techniques; hence, we test the following hypothesis

H_0: Adaptive threshold determination via time-varying selection for the estimator of the inverse Hill statistic is a universally applicable approach to improve the stability and robustness of tail estimates compared to methods based on threshold selection procedures determined by Hill plots, the mean excess life function, or historical or adaptive test sequence data analysis (TSDA) associated with the forwardstop algorithm.

Finally our study presents three principal contributions: first, it introduces a dynamic, time-varying threshold selection mechanism for extreme value analysis utilising the inverse Hill statistic, thereby improving the stability and reliability of tail risk estimation in volatile energy markets; second, it applies weighted nonlinear least squares for robust estimation of Generalised Pareto Distribution parameters in non-stationary contexts, addressing the shortcomings of maximum likelihood and Bayesian methods; and third, it offers a thorough empirical evaluation demonstrating that these techniques produce more precise and resilient Value-at-Risk and Expected Shortfall estimates compared to traditional static or semi-static methods.

2. Literature review

The Generalised Pareto Distribution is a fundamental component of extreme value theory and has been widely used in several fields, including environmental sciences, finance, and energy markets. Early foundational research demonstrated that, despite the theoretical appeal of maximum likelihood estimation, alternative estimators such as the method of moments (MOM) and probability-weighted moments (PWM) frequently outperform MLE in finite samples, especially when sample sizes exceed moderate thresholds (Hosking and Wallis, 1987). This understanding led to a wide variety of practical research that used GPD modelling and econometric filtering methods to get better tail estimates.

In financial markets, a prevalent modelling approach is filtering returns using Autoregressive Moving Average-GARCH (ARMA–GARCH)-type models before fitting the GPD to the residual. Evidence from the Johannesburg Stock Exchange shows that this method makes it easier to estimate extreme returns and tail-related risk measures. Nevertheless, stationary GPD models outperform typical volatility models in predicting extreme movements, as emphasised by Sigauke et al. (2014). Similarly, studies of the Ghanaian and South African equity markets, such as those by Nortey et al. (2015) and Chikobvu and Chifurira (2015), yield comparable results. Both the GPD and the generalised extreme value (GEV) distributions deliver superior tail risk estimates compared to the normal distribution, especially in the lower tail. Taken together, these results confirm the efficacy of GPD-based approaches for modelling financial extremes. However, the presumption of stationarity in several initial applications limits their ability to capture the dynamic behaviour of tails. To tackle this issue, several works have enhanced GPD-based models to include time-varying dynamics. For example, Sigauke et al. (2012) propose an ARMA–EGARCH–generalised single Pareto model for estimating extreme electricity demand, yielding improved tail quantile estimates compared to volatility models. However, EGARCH-based designs assume a static error distribution and struggle to adapt to regime shifts or changes in tail behaviour. To address these constraints, Makatjane and Tsoku (2022) provide a SARIMA–GAS–GEV charter that includes Bayesian bootstrapping and density-prediction backtesting. This work highlights both the inherent flexibility and the considerable complexity of non-stationary

extreme value distribution. Their results show that enhancing these models requires careful definition not only of time-varying tail parameters but also of the bulk distribution, making both the computational and conceptual work difficult.

Nonetheless, selecting an appropriate threshold remains a major challenge in peak-over-threshold models. Researchers often employ graphical and empirical approaches, though these are not particularly reliable or objective. Automated, data-driven methods were proposed to address these problems. Hence, Zoglat et al. (2013) juxtapose automated threshold selection methods with enhanced square-error techniques and propose a multiple-threshold strategy, demonstrating superior performance relative to conventional methods. Magadia (2010) further discusses the problems with using standard threshold selection in the presence of volatility clustering and suggests a self-exciting point process architecture as a better option. These results highlight the susceptibility of POT-based inference to threshold selection and the need for more resilient, adaptable methodologies. Additional contributions concentrate on enhancing tail estimates by tackling reliance and asymmetry in returns; therefore, Maghyereh and Al-Zoubi (2008) show that employing an Autoregressive Fractionally Integrated Asymmetric Power Autoregressive Conditional Heteroscedasticity (AR-FIAPARCH) model to pre-filter stock returns makes it easier to follow the independently and identically distributed (i.i.d.) assumptions needed for EVT, which leads to better tail quantile estimates. Additional evidence from volatility-based research indicates that idiosyncratic volatility significantly influences the prediction of extreme returns, accentuating the need for models that explicitly incorporate tail dynamics rather than relying solely on conditional variance (Switzer et al., 2017). The GPD and GEV distributions have effectively been used in environmental and energy-related extremes, extending beyond financial markets; therefore, Sikhwari et al. (2022) use r-largest order statistics and MLE to predict severe rainfall, illustrating the utility of EVT for long-term risk assessment. On the other hand, Sigauke and Bere (2017) use smoothing splines to model time-varying thresholds in the context of power consumption, integrating temperature as a covariate, and determine that the Weibull class offers a better fit in both stationary and non-stationary scenarios. Masilo and Makatjane (2025) also used block minimum GEV modelling to look at high power losses in South Africa. The authors found that the Weibull distribution remains strong across seasonal regimes.

The literature together illustrates significant advancements in extreme value modelling while also exposing enduring constraints. These include dependence on stationary-tail assumptions, the susceptibility of MLE to limited sample sizes and complex models, and persistent subjectivity in threshold determination. Non-stationary and Bayesian extensions provide greater flexibility; nonetheless, they often pose significant computational and modelling challenges. In this context, the current research offers a time-varying GPD model estimated using weighted nonlinear least squares, integrated with an adaptive threshold selection method. This method immediately fills the gaps by stabilising the distribution, reducing the likelihood of crashes, and providing a flexible yet manageable structure for predicting dynamic tail risk. Prior research advanced extreme value modelling in finance and energy, yet gaps remain. Stationary GPD models rely on subjective threshold selection and often fail to capture dynamic tail behaviour. Non-stationary approaches, such as Bayesian and ARMA–EGARCH–GPD models, can be complex and computationally demanding. No framework currently combines automatic, stable time-varying threshold selection, robust estimation of non-stationary GPD parameters, and application to financial instruments in the South African energy sector, such as the FTSE/JSE Oil, Coal, and Gas Index. To address these gaps, our study proposes weighted nonlinear least squares with adaptive threshold selection. This delivers a practical and sound framework for modelling dynamic tail risk.

3. Methodology

The data used in this study comprises a five-business-day financial time series of the Johannesburg Stock Exchange (JSE) FTSE/JSE Oil, Coal and Gas Index (FTSE/JSE-J500) for the period from 08 January 2012 to 04 February 2025. This data is obtained from the Johannesburg Stock Exchange (JSE). It is worth noting that this index is a capitalisation-weighted benchmark that tracks the performance of South African companies engaged in the exploration, production, and distribution of oil, coal and gas, serving as a key indicator of the energy sector's market dynamics within the JSE. To

avoid exchange-rate fluctuations, the index is kept in its original currency. And, assuming that the returns are represented as $r_t = 100\left[\frac{\ln(X_t)-\ln(X_{t-1})}{\ln(X_{t-1})}\right]$ Bee and Trapin (2018) declared that r_t is the depreciation percentage of the energy returns between $t-1$.

The Generalised Pareto Distribution and peaks over threshold

The peaks-over-threshold approach considers only observations that exceed or fall below a specified threshold (Pickands, 1975), as cited by Naradh et al. (2021). In this study, the inverse Hill statistic for choosing an optimal threshold for the generalised Pareto distribution is applied by first looking at the integrated square error (ISE) between the exponential density h_γ and its parametric estimator $h_{\hat{\gamma}k}$ and this ISE is estimated as

$$ISE(k) = \int \left\{h_\gamma(x) - h_{\hat{\gamma}k}(x)\right\}^2 dx = \frac{1}{2\gamma} - \frac{2}{\gamma+\hat{\gamma}_k} + \frac{1}{2\hat{\gamma}_k}, \tag{0.1}$$

where, $h_\gamma(x) = \frac{1}{\gamma}\exp -\,{}^{1}\!/_{\gamma}$ and $\hat{\gamma}_k$ is the Hill estimator. Since the first term of the ISE is constant, choosing k is not affected by it; hence, the second term of the integrated squared error (ISE) remains unknown, while the final term is well-defined. Subsequently, the ISE cannot be minimised directly; instead, an exponential approximation is used to estimate and minimise the mean of the ISE and test the hypothesis that the Hill estimator follows a Gamma distribution in this study. Under this assumption, the mean integrated squared error (MISE) is explicitly calculated, and it is observed that, under the exponential approximation, MISE is a decreasing function of the parameter k, which is given by

$$MISE(k) = -\frac{1}{2\gamma} = E_{H_0}\left[ISE(k)\right] - \frac{1}{2\gamma}C(k) + \frac{k}{2(k-1)\gamma'} \tag{0.2}$$

where, $C(k) = 2\exp(k)k^k\Gamma(1-k,k)$ and $\Gamma(a,n) = \int_b^\infty t^{a-1}e^{-t}dt$ denotes the upper incomplete gamma function. When $C(k) \approx 1 + {}^{1}\!/_{(4k)}$ Model (1.2) converges fast, to obtain Model (1.3) as

$$\lim_{k\to\infty} E_{H_0}\left[\frac{2}{\gamma+\hat{\gamma}_k}\right] = \frac{1}{\gamma} = E_{H_0}\left[\frac{k-k}{k\gamma_k}\right]. \tag{0.3}$$

As this happens, an estimator for the first term in Model (1.3) is provided under H_0. However, owing to its high variability for small k, the study instead aims to find an estimator of the form $\omega/\hat{\gamma}_k$ for some ω depending on k that minimises the mean square error (MSE), and approximates its MSE in the following manner

$$E_{H_0}\left[\left(\frac{\omega}{\hat{\gamma}_k} - \frac{2}{\hat{\gamma}_k+\gamma}\right)^2\right] \approx \frac{\omega^2k^2}{\gamma^2(k-1)(k-2)} - \frac{2\omega k}{\gamma^2(k-1)} + \frac{1}{\gamma^2}. \tag{0.4}$$

Schneider et al. (2021) declared that this approximation depends on similar functions $C(k)$, which quickly becomes constant. The MSE in Model (1.4) is minimised for $\omega = {}^{(k-2)}\!/_{k}$, thus, the inverse Hill statistic is obtained by

$$IHS(k) = {}^{1}\!/_{2\hat{\gamma}_k} - {}^{k-2}\!/_{\hat{\gamma}_k k} = {}^{4-k}\!/_{2\hat{\gamma}_k k} \tag{0.5}$$

to estimate $MISE(k) - (2\gamma)^{-1}$, and the sample fraction is selected via minimising IHS, i.e, $\hat{k}_{IHS} = \arg\min_{1<k<n}\left\{IHS(k)\right\}$ because IHS exhibits high fluctuations for small values of the parameter

k, this makes the automated selection inconsistent or produces unreliable estimates. To control this problematic behaviour, Model (1.5) is being smoothed in this study; more specifically, $E\left[IHS\right]$ is estimated by considering the following regression problem in Model (1.6)

$$IHS\left(k\right)=E\left[IHS\left(k\right)\right]+\sigma\in_k,k=1,\cdots,n, \tag{0.6}$$

where, $\sigma>0$ and $E\left[\in_k\right]=0$. Due to the structure of the Hill estimator, the random variables $\in_k$ are highly dependent, which needs to be taken into account in the estimation process. Nonetheless, a natural cubic spline smoothing procedure is applied in this study to obtain a smooth estimator for the expectation of IHS.

Time-heterogeneous Generalised Pareto Distribution

For the non-stationary generalised Pareto distribution, the study allows both the scale and the shape parameters to evolve. By including time-dependent parameters, the aim is to account for heightened market crashes and extreme volatility observed during such periods, and define the scale and shape parameters as $\sigma\left(t\right)=\exp\left(\sigma_0+\sigma_1t\right)$ and $\xi\left(t\right)=\xi_0+\xi_1t$ where, $\sigma_{i,j}$ and $\xi_{i,j}$ $,i=0,j=1$ govern the evolution of the scale and shape parameters, respectively, over time. Finally, the probability density function for a time-heterogeneous GPD according to Singirankabo and Iyamuremye (2022) and Maposa and Cochran (2017) is given by

$$f\left(v\middle|\sigma\left(t\right),\xi\left(t\right)\right)=\frac{1}{\sigma\left(t\right)}\left(1+\frac{\xi\left(t\right)v}{\sigma\left(t\right)}\right)^{-\frac{1}{\xi\left(t\right)}-1},v>0 \tag{0.7}$$

where the following condition must be satisfied $\sigma\left(t\right)>0,\forall t$. Since $\sigma\left(t\right)$ represents the scale (spread) of the distribution, it must always be positive, and a common parameterisation is $\sigma\left(t\right)=\exp\left(\sigma_0+\sigma_1t\right)$ to ensure it remains positive. For the shape parameter, $1+\frac{\xi\left(t\right)v}{\sigma\left(t\right)}>0,\forall v>0$ constraint is imposed to ensure that the term inside the power function remains positive.

Risk measures

This section of the study presents the methodology used to estimate the downside risk using VaR and ES.

Value-at-risk

Value-at-risk is a common measure of extreme risks, and the GPD is used in this study to approximate this measure. In particular, using Model (1.7), the value-at-risk at time t with the confidence level α is obtained by

$$VaR_\alpha=u_t+\frac{\sigma_t}{\xi_t}\left[\left(\frac{1}{1-\alpha}\right)^{\xi_t}-1\right], \tag{0.8}$$

for a given threshold u_t. In this case, VaR_α presents the 100_α quantile to quantify tail risk in several fields, with p close to 1.

Expected shortfall

In contrast to VaR, ES measures the riskiness of an instrument by considering both the size and likelihood of losses above a particular threshold (Basel III, 2019). The expected shortfall gives the expected size of return that exceeds VaR, i.e., for a probability level α, thus, ES for the GPD in Model (1.7) is given by

$$ES_\alpha=\frac{VaR_\alpha}{1-\xi_t}+\frac{\hat{\sigma}_t-\hat{\xi}_tu_t}{1-\hat{\xi}_t}. \tag{0.9}$$

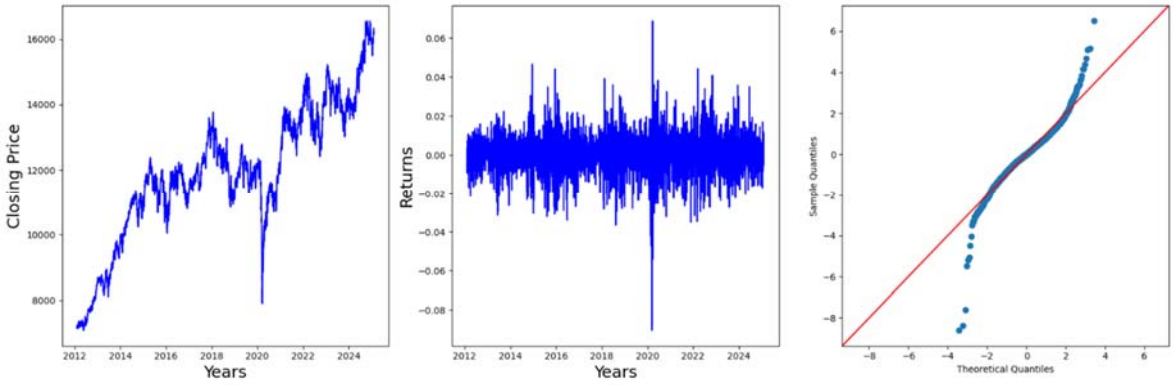
provided that $\hat{\xi}_t<1$ it ensures finite risk measures.

Empirical results

The five-day returns on the FTSE/JSE-J500 energy index, shown in Figure 1, indicate that energy stock prices are not normally distributed. The first panel shows both rising and falling patterns,

as well as seasonal changes. Periods of volatility clustering are driven by market panic from the European debt crisis of 2010, the Shanghai stock market crisis of 2015/2016, the COVID-19 pandemic, and geopolitical and economic factors. The financial market experienced some of the largest stock-return losses, as shown in Figure 1. This figure also suggests that adding conditional heteroscedasticity could help; thus, two main concerns arise: (1) the source of volatility, and (2) the predictability of these losses. The first theory proposes that random shocks to the real economy increase future volatility. The second theory suggests that such shocks mainly lead to heightened volatility during downturns, rather than causing immediate, dramatic losses. The quantile-quantile (Q-Q) plot in the right panel shows that the tails of the marginal distribution are not linear. Table 1 confirms this with kurtosis greater than 3 and negative skewness. This indicates a lack of symmetry in the FTSE/JSE-JI006 energy stock returns, leading to negatively skewed innovations.

Figure 1. FTSE/JSE-J500 stock returns



Source: Authors own computations

The mean of FTSE/JSE-J500 returns in Table 1 is positive. The deduction here is that the overall energy stock prices for this sector are increasing during the period under study. The magnitude of the average return is very small at 0.031%. Finally, the normality tests all reject the null hypothesis, concluding that the returns are not normally distributed, agreeing with the results of the Q-Q plot in Figure 1.

Table 1. Descriptive statistics for returns series

	Central Tendency Measures				Normality Tests		
	Mean	**Std dev**	**Skewness**	**Kurtosis**	**SW**	**KS**	**Jb**
Returns	0.000314	0.016802	-0.23981	16.61118	0.87671(0.001)	0.47295(0.001)	22344(0.001)

Key: SW is Shapiro-Wilk; KS is Kolmogorov-Smirnov, and lastly JB is the Jarque-Bera.

Threshold selection and performance

For the convenience of presentation, the data is now rescaled as $r_t \to 100 * r_t$, and this led the study to produce all analogous results for negative returns by considering the following relation $\min\{x_1,\cdots,x_n\} = \max\{-x_1,\cdots,x_n\}$. For a fixed threshold, the ForwardStop algorithm is employed; for a time-varying threshold, the proposed inverse Hill statistic is used. Moreover, to generalise the results, mean excess life and the Hills plot are used as benchmarks to quantify exceedances above the chosen thresholds for long and short trading positions. The analysis of threshold estimation methods for returns in Table 2 highlights key differences in accuracy and reliability. The Inverse Hill Statistic is the most accurate method, with an extremely low MSE (0.00001) and near-zero bias (0.00001), indicating high reliability. The Hill Plot also performs well, exhibiting a low MSE (0.01180) and minimal bias (0.00011), indicating stability. In contrast, the ForwardStop method shows the highest bias (0.10629) and a relatively high MSE (0.01129), making it less reliable for precise threshold estimation. The Mean

Excess Life method, though stable, has the highest MSE (0.01851) and a moderate bias (0.00361), suggesting some inefficiency in capturing extreme values.

Table 2. Performance of threshold selection

Returns				
Threshold Method	**Threshold**	**MSE**	**BIAS**	**Comments**
ForwardStop	0.09051	0.01129	0.10629	Highest bias, less reliable
Inverse Hill Statistic	0.01217	0.00001	0.00001	Extremely accurate, near-zero bias
Mean Excess Life	0.01578	0.01851	0.00361	Higher MSE, slightly biased
Hills Plot	0.01578	0.01180	0.00011	Low MSE, stable

Estimation of a time-heterogeneous Generalised Pareto Distribution

The time-heterogeneous Generalised Pareto Distribution (GPD) is fitted to exceedance returns using a weighted nonlinear least-squares likelihood-moments algorithm. The optimal threshold is determined using the dynamic inverse Hill statistic, yielding a value of 0.01217 with 311 exceedances. Time hierarchies, including daily, weekly, monthly, quarterly, half-yearly, and yearly intervals, are incorporated as covariates to capture evolving risk dynamics. The estimated scale parameter is decomposed into a constant and a time-varying component, with $\hat{\sigma}_0 = 0.01717305$ and a very small time-varying scale coefficient $\hat{\sigma}_1$ is approximately 0. The low standard error associated with the scale parameter indicates stable estimation and suggests that extreme negative returns occur with notable frequency, implying that investors are exposed to substantial downside risk. Similarly, the negligible shape parameter is specified as time-varying, with an estimate $\hat{\xi}_1$ which is approximately 0, while the shape parameter is denoted as $\hat{\xi}_0$ confirming a heavy-tailed distribution, indicating that extreme losses are more probable than under a normal distribution. While the inclusion of time-varying covariates is intended to capture changing market conditions, such as periods of heightened volatility during the 2015–2016 Shanghai stock market crash and the COVID-19 pandemic, and the near-zero estimates of time-varying scale and shape parameters suggest that, in this case, time variation in the tail behaviour is limited. This finding implies that, although the modelling framework allows for dynamic risk, the empirical evidence points to relatively stable tail characteristics over time. Nonetheless, the heavy-tailed nature of the distribution remains important for risk management, as it highlights the persistent likelihood of extreme market movements. These results are consistent with Chinhamu et al. (2015), who also report a positive shape parameter corresponding to a Type II GPD (Fréchet class). Further support is provided by the approximate linearity observed in the Q–Q plot, indicating a good fit of the GPD to the exceedance data.

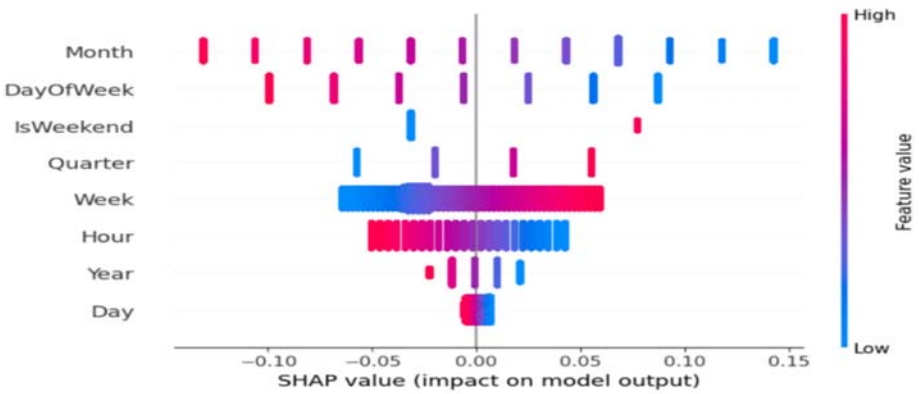
Table 3. Parameter estimates of the fitted time-heterogeneous GPD

Threshold	0.01217
Exceedances	311
$\hat{\sigma}_0$	0.01717305
$\hat{\sigma}_1$	0.0000005747881
$Se(\hat{\sigma}_0)$	0.001030915
$Se(\sigma_1)$	0.0000003599534
$\hat{\xi}_0$	0.01381969
$\hat{\xi}_1$	0.00000414034
$Se(\hat{\xi}_0)$	0.001748085
$Se(\hat{\xi}_1)$	0.0000003599534

Feature importance and stability of time-varying-GPD

Figure 2 shows the use of temporal hierarchies, ranging from hourly to year-long intervals, to better forecast FTSE/JSE-J500 return risk. Using SHAP, the model identifies which attributes most influence return forecasts. The month variable has the most significant positive effect, suggesting seasonal trends. The day-of-the-week variable also affects returns. Returns are generally higher from Tuesday through Friday, lower on Mondays, and lowest on weekends, as indicated by the IsWeekend variable. This distinction is important: weekend trading, despite the absence of official closures, features lower liquidity and market depth, leading to calmer price dynamics. Equity market activity is lowest on Mondays and greatest on the final two working days. In the South African energy stock market, weekends are associated with lower projected returns due to volatility. Strohsal et al. (2019) describes the Monday effect, while Atsin and Ocran (2015) show that returns on other days tend to be positive on average, but Mondays are significantly negative.

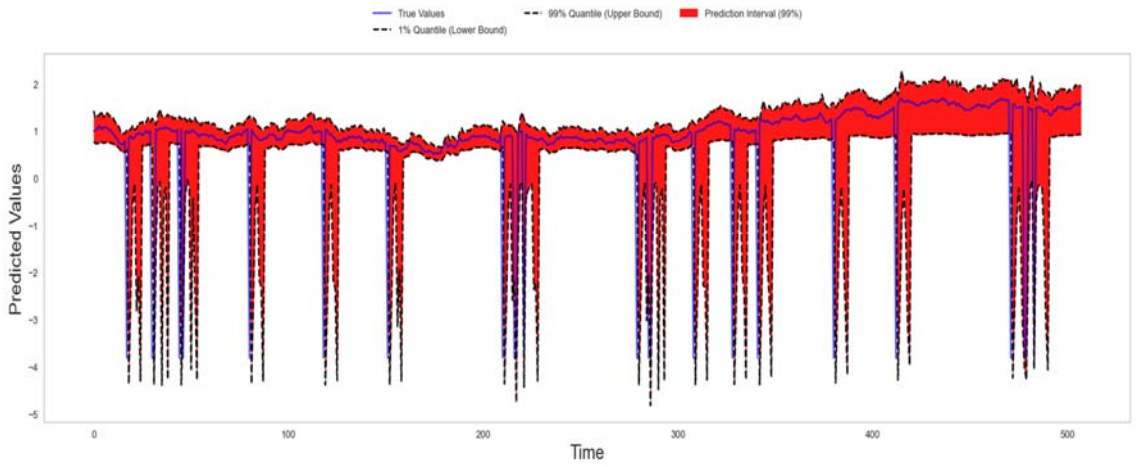
Figure 2: SHAP values for feature importance



Even while patterns may be seen every week, there are also noticeable impacts daily. Peak trading times correlate with better performance, suggesting that operational scheduling or demand-response dynamics are at play in the hour-of-day impact on returns. On the other hand, there is no correlation between the day of the month and the outcome, albeit a small annual reduction could suggest a shrinking market or the existence of external restraints. These insights into time improve the performance of the time-varying GPD by highlighting observable patterns that inform its predictions. By identifying and validating temporal dependencies, the distribution can generalise more effectively, adjust to changes in regimes, and provide more precise predictions of negative risk under uncertain and dynamic market situations. The tail risk model may better explain the peculiar risk-reward dynamics of the bitcoin market since it takes these time-based impacts into account directly. This evidence is supported by the prediction interval widths reported in Figure 3.

The mean prediction interval width (MPIW) of 1.0277, together with the 99% prediction interval illustrated in Figure 3, provides a clear indication of the model's predictive accuracy and the uncertainty surrounding its forecasts. The relatively narrow MPIW suggests that the model generates precise predictions while appropriately accounting for inherent variability. Furthermore, the true values, depicted by the blue line, largely remain within the 99% prediction interval (shaded red), demonstrating the model's capacity to capture the underlying downside tail risk effectively. These results indicate that our estimated time-varying GPD modelling approach is robust and capable of producing reliable out-of-sample forecasts, an essential feature for both economic decision-making and market risk management.

Figure 3. Assessment of model stability



Forecasting and backtesting VaR and ES estimates

Upon confirming the suitability of the calculated GPD model for robust out-of-sample forecasting under uncertainty, we proceed to compute Value-at-Risk (VaR) and Expected Shortfall (ES). The system is deployed on AWS, leveraging EC2 instances for resource-intensive computations and a stable environment for model updates, while Lambda functions enable serverless, event-driven execution for real-time prediction of downside risk. This hybrid architecture ensures both computational efficiency and responsiveness, allowing the energy sector under study to monitor extreme risks continuously and react promptly to new market or operational data. The sufficiency of these risk indicators is explicitly evaluated by combined backtesting, using a dynamic quantile test that concurrently assesses VaR and ES. This collaborative testing method facilitates an exhaustive validation of tail risk predictions by considering both the frequency of exceedances and the severity of severe losses, thereby offering a more rigorous evaluation of model efficacy. Table 4 and Table 5 present distributional estimates of value-at-risk and expected shortfall, the number of VaR and ES violations, and the results of the Kupiec likelihood ratio test and the Christoffersen test applied to the FTSE/JSE-JI006 energy stock price index. All VaR and ES calculations, along with their respective statistical tests, are conducted at standard confidence levels: 0.1%, 1%, and 5% for long positions, and 95%, 99%, and 99.9% for short positions.

Table 4. Performance of VaR and ES for the losses

Threshold Method	Trade position	VaR	ES
Inverse Hill Statistic	0.1%	0.01213564	-0.02602860
	1%	0.01182465	-0.02637414
	5%	0.01040384	-0.02795282
	95%	-0.10778901	-0.15927821
	99%	-0.18870792	-0.24918810
	99.9%	-0.32964663	-0.40578667

The results obtained from the Inverse Hill Statistic method provide critical insights into the extreme risk dynamics within the FTSE/JSE-J500 energy market in South Africa. These results are reported in Table 4, and at lower quantiles (0.1%, 1%, and 5%), the value-at-risk estimates remain positive, suggesting that moderate losses are expected within typical market fluctuations. However, at higher quantiles (i.e., 95%, 99%, and 99.9%), the VaR values turn increasingly negative, reflecting severe downside risks. The expected shortfall values further reinforce this perspective, indicating that in the event of a breach beyond the VaR threshold, the average losses are significantly larger. Notably,

at the 99.9% confidence level, the VaR estimate of -0.3296 advocates for an extreme loss scenario, with the ES estimate of -0.4058 implying that, in the worst 0.1% of cases, the expected loss will be even greater.

This approach is different from that of He et al (2021) because these authors have used a stationary GPD while the current study uses a time-heterogeneous GPD. He et al (2021) offered a random weighted bootstrap method for interval estimation of VaR, demonstrating that their approach effectively covers the true VaR in both insurance and finance applications, while in this study, the truncated Wild bootstrap method is applied. The pronounced losses at higher quantiles advise that extreme downturns in this sector could lead to significant capital drawdowns, reinforcing the necessity for robust risk management strategies. From a financial stability perspective, such levels of risk may have broader implications for investment decisions, as market participants must account for potential black swan events that could severely disrupt returns.

Table 5. Backtesting of VaR and ES for energy returns

		Kupiec test						**Number of Violations**					
	Proportion	0.1%	1%	5%	95%	99%	99.9%	0.1%	1%	5%	95%	99%	99.9%
VaR	**Losses**	0.3768	0.178	0.858	0.77	0.0382	0.892	3	26	123	112	26	3
	Christoffersen test							**Number of Violations**					
	Losses	0.7596	0.0086	0.594	0.687	0.657	0.7596	1	19	126	111	21	2
	Dynamic Quantile												
ES	**Losses**	0.3834	0.0949	0.594	0.217	0.358	0.7596						

The statistical robustness of the VaR estimates across several confidence levels is confirmed by both the Kupiec test and the Christoffersen test. The dynamic quantile is used to evaluate the robustness of the ES estimates. The findings show that both VaR and ES estimates from the time-heterogeneous GPD are good, with p-values mostly confirming their dependability in risk estimation. Huang et al. (2014), in their analysis of generalised hyperbolic distributions and value-at-risk assessment for the South African mining index, also discovered significant VaR violations at 5% and 95% levels identical to those reported in this study. The model shows better performance in grasping extreme losses at the 0.1% and 99.9% thresholds, where just three violations are seen at both levels, hence improving its risk assessment capacity. Chinhamu et al (2014), particularly at the 0.1% and 99% levels, also found this low number of violations we had found in this study. The model effectively reflects significant market volatility, implying a more conservative attitude to risk estimation for the VaR. The dynamic quantile test's generally high p-values confirm the strength of the model in evaluating risk for both long and short trading positions. These results are also in line with those found by Chinhamu et al (2015) in their study of extreme risk, value-at-risk and expected shortfall in the gold market, but these authors had used the bootstrap backtesting procedure found in McNeil and Frey (2000), with and without bootstrapping.

Both the Kupiec test and the Christoffersen test show that the VaR model is statistically strong at a range of confidence levels. Furthermore, the use of dynamic quantiles to check how strong the ES estimates are gives robust modelling outcomes, confirming that the ES from the fitted time-heterogeneous GPD is a precise estimate of downside risk, especially in the tails of the distribution. The results suggest that the models are effective, and the p-values largely support their use for estimating risk. Huang et al. (2014) looked at the generalised hyperbolic distribution and value-at-risk for the South African mining index. They also found large VaR violations at the 5% and 95% levels, just like we did in this study. The model does a better job of noticing extreme losses at the 0.1% and 99.9% thresholds, where there are only three violations at both levels. This means it better assesses risk. Chinhamu et al. (2014) also found this low number of violations, especially at the 0.1% and 99% levels. The model accurately reflects significant market volatility, which means that the VaR is more conservative when it comes to estimating risk. The dynamic quantile test generally shows strong p-values, implying that the model is effective at estimating risk for both long and short trading positions. We also observed results that are similar to those of Chinhamu et al. (2015), who studied extreme risk, value-at-risk, and

projected deficit in the gold market. However, these authors employed the backtesting method from McNeil and Frey (2000), both with and without bootstrapping.

4. Discussion

The energy sector operates within a highly volatile and interconnected financial environment, where risk is shaped by macroeconomic conditions, geopolitical developments, regulatory interventions, and supply–demand dynamics. External shocks, including abrupt commodity price movements, exchange rate fluctuations, and global crises, amplify uncertainty and expose investors and policymakers to substantial downside risk. Within this context, accurate modelling of extreme losses is essential for informed decision-making, regulatory compliance, and systemic risk monitoring. This study contributes to this objective by modelling Value-at-Risk and Expected Shortfall for the FTSE/JSE-JI006 energy sector using a time-varying Generalised Pareto Distribution, an area that has received limited attention in the South African financial literature. The empirical findings demonstrate that incorporating time-heterogeneous thresholds substantially improves the accuracy of tail risk estimates across multiple time horizons. Allowing tail behaviour to evolve dynamically enables the model to capture periods of heightened instability more effectively, particularly during episodes of extreme market stress such as the COVID-19 pandemic and the 2015–2016 Shanghai stock market crash. Backtesting evidence from the Kupiec and Christoffersen tests provides consistent support for the adequacy of the proposed model, with fewer violations observed at extreme confidence levels for both long and short positions. These results extend earlier findings, such as those of Chinhamu et al. (2015), by showing that relaxing stationary tail assumptions yields meaningful improvements in risk forecasting performance, particularly in volatile and emerging markets.

From an economic perspective, the results reinforce the importance of accounting for fat-tailed and asymmetric return distributions when assessing risk in energy-related assets. Conventional models grounded in normality assumptions tend to underestimate extreme losses, leading to misleading risk assessments and potentially insufficient capital buffers. The leptokurtic nature of returns observed in the South African energy sector is consistent with earlier evidence (Korkpoe and Junior, 2018; Naradh et al., 2022). This study advances the literature by demonstrating that a dynamic threshold specification enhances sensitivity to changing market conditions, thereby improving the reliability of risk measures during turbulent periods. For investors and portfolio managers, this translates into more prudent capital allocation, improved hedging decisions, and a reduced likelihood of excessive risk-taking during episodes of market stress.

Additional insight into the drivers of tail risk dynamics is provided by the SHAP (Shapley Additive exPlanations) analysis. The SHAP results reveal that temporal features play a dominant role in explaining variations in downside risk, highlighting the importance of calendar effects and time hierarchies in financial return behaviour. Certain periods exert a disproportionate influence on model predictions, reflecting shifts in market liquidity, trading intensity, and information flow. By quantifying the relative importance and directional impact of these temporal components, the SHAP analysis enhances the interpretability of the model and provides economic intuition regarding when and why tail risk intensifies. This interpretability is particularly valuable for risk managers and policymakers, as it promotes transparency in model-based decision-making and supports more targeted risk mitigation strategies. The reliability of the model's uncertainty quantification is further supported by the prediction interval analysis. The mean prediction interval width (MPIW) of 1.0277423, together with the 99% prediction interval, indicates that the model delivers relatively precise forecasts while adequately capturing uncertainty in extreme outcomes. The fact that observed returns largely remain within the predicted bounds suggests that the model effectively characterises downside tail risk in an out-of-sample setting. Achieving this balance between sharpness and coverage is crucial for practical risk management, as excessively wide intervals limit decision usefulness, while overly narrow intervals risk underestimating extreme losses.

The policy and regulatory implications of these findings are substantial. Accurate estimation of VaR and ES lies at the core of prudential regulation, stress testing, and capital adequacy assessments under Basel III and related supervisory standards. The superior performance of the time-varying GPD suggests that regulators and supervisory authorities could benefit from adopting dynamic tail risk

models when evaluating the resilience of energy-sector portfolios. Such models offer earlier warning signals of systemic vulnerability and enhance the credibility of stress-testing exercises, thereby strengthening macroprudential oversight. More broadly, for policymakers concerned with energy market stability, the ability to detect and monitor periods of elevated downside risk is essential for timely intervention. By integrating dynamic tail modelling with interpretable feature attribution and reliable uncertainty quantification, this study provides a comprehensive and policy-relevant perspective on the evolution of extreme risks in energy markets.

5. Conclusion

This study introduces a time-varying Generalised Pareto Distribution to better evaluate downside risk in the FTSE/JSE-J500 energy stock price index. An inverse Hill statistic is suggested for threshold selection and benchmarked against conventional techniques, showing superior accuracy with minimal bias and the lowest mean squared error. The study successfully fits the model to the data using weighted nonlinear least squares, allowing it to capture the evolving nature of extreme risk more accurately. The inclusion of temporal hierarchies enhances risk assessment by capturing volatility patterns over multiple time scales, while the estimated shape parameter confirms a heavy-tailed distribution, emphasising the high likelihood of extreme negative returns. Backtesting results demonstrate that the proposed model reliably identifies severe financial downturns and effectively accounts for the dependence between extreme events.

Despite these strengths, the methodology has some limitations. First, extreme, unprecedented market shocks - such as sudden regulatory changes or economic crises- may temporarily challenge the predictive accuracy of the model. Second, the approach does not explicitly incorporate macroeconomic or sector-specific covariates, which could further refine extreme risk predictions. Third, although the system has been implemented in real time on AWS EC2 and Lambda instances, extremely high-frequency data or simultaneous application to multiple indices may increase computational demands. Finally, comparisons with machine learning–based models (e.g., ESRNN, LSTM, or VAE frameworks) were not included, leaving an open question regarding relative performance under alternative modelling paradigms.

Future work could address these limitations by integrating macroeconomic and energy-sector covariates, systematically comparing performance with machine learning–based approaches, extending the methodology to other energy markets or renewable-heavy portfolios, and investigating stress-testing procedures under extreme external shocks. These steps would further enhance the practical applicability and robustness of adaptive, time-varying tail risk models in volatile financial markets.

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